**Goodwillie, Thomas G.**
**Calculus. I: The first derivative of pseudoisotopy theory.** (English) Zbl 0741.57021
*K*-Theory 4, No. 1, 1-27 (1990).

A $k$-connected map $Y \to X$ of manifolds induces a $(k-2)$-connected map $\mathcal{P}(Y) \to \mathcal{P}(X)$ of the associated stable smooth pseudoisotopy spaces. The main result of the paper describes the $(2k-3)$-homotopy type of $fiber(\mathcal{P}(Y) \to \mathcal{P}(X))$. Let $\Lambda(X) = Maps(S^1, X)$ denote the free loop space and $\Lambda(Y \to X) = Y \times_X \Lambda(X)$. The inclusion of constant loops $X \to \Lambda(X)$ induces an inclusion $Y \to \Lambda(Y \to X)$. The $(2k-3)$-homotopy type of $fiber(\mathcal{P}(Y) \to \mathcal{P}(X))$ is the same as that one of $fiber(\Omega^2 Q(\Lambda(Y \to X)/Y) \to \Omega^2 Q(\Lambda(X)/X))$ with $Q = \Omega^\infty \Sigma^\infty$. As a corollary the author determines the $(2k - 3)$-homotopy type of $fiber(A(Y) \to A(X))$, Waldhausen's relative $K$-theory of topological spaces. Using the theory of calculus of functors the proof reduces to a geometric analysis of the map $P(N) \to P(M)$ of the unstable pseudoisotopy spaces induced by the inclusion $N \subset M$, where $M$ is obtained from $N$ by attaching a handle. The calculus of functors, developed by the author in the past decade, is a theory particularly suited for stable range calculations and has had remarkable applications. The paper starts with a concise treatment of the differentiation and the derivative of homotopy functors (higher derivatives and the powerful theory of analytic functors are deferred to a later paper). The derivative of $X \mapsto Q(Map(K, X)_+)$, $K$ a finite complex, is determined in Section 2. This example is of special interest in view of the connection of $Map(S^1, X)$ to algebraic $K$-theory. The main result is phrased as calculation of the derivative of the functor $X \mapsto \mathcal{P}(X)$, which determines the $(2k - 3)$-homotopy type of $fiber(\mathcal{P}(Y) \to \mathcal{P}(X))$. Its proof constitutes Section 3.

Reviewer: R. Vogt (Osnabrück)

**MSC:**

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|---|---|
| 57R52 | Isotopy in differential topology |
| 19D10 | Algebraic $K$-theory of spaces |
| 55P65 | Homotopy functors in algebraic topology |

Cited in **16** Reviews
Cited in **59** Documents

**Keywords:**

differentiation and derivative of homotopy functors; stable smooth pseudoisotopy spaces; free loop space; Waldhausen's relative $K$-theory of topological spaces; calculus of functors; unstable pseudoisotopy spaces; attaching a handle; algebraic $K$-theory

**Full Text:** DOI

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