Slodkowski, Zbigniew

Holomorphic motions and polynomial hulls. (English) Zbl 0741.32009

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Let E be a subset of the complex line $\mathbb{C}$. A holomorphic motion of E in $\mathbb{C}$, parametrized by the unit disc D, is a map $f : D \times E \to \mathbb{C}$ such that (a) for any fixed $\omega \in E$, the map $z \to f(z, w)$ is holomorphic on D; (b) for any fixed $z \in D$, the map $w \to f(z, w)$ is an injection; (c) $w \to f(0, w)$ is an identity on E.

The author gives an affirmative answer of the two questions posed by Sullivan and Thurston:

Theorem 1.3. Every holomorphic motion $f : D \times E \to \mathbb{C}$ of an arbitrary subset E of $\mathbb{C}$ can be extended to a holomorphic motion $F : D \times \mathbb{C} \to \mathbb{C}$ of the whole of $\mathbb{C}$, parametrized by the same unit disc.

Theorem 1.4. Let $f(z, w) = f_z(w)$ be a holomorphic motion of a subset $E \subset \mathbb{C}$, parametrized by $z \in D$. Then, for every point a outside E, there is a holomorphic map $g : D \to \mathbb{C}$ such that

(i) $g(0) = a$ and

(ii) $g(z) \notin f_z(E)$ for every $z \in D$.

In his proofs the author uses the relation between holomorphic motions and polynomial hulls.

Reviewer: S.M.Ivashkovich (Bochum)

MSC:

32E20 Polynomial convexity, rational convexity, meromorphic convexity in several complex variables
30E25 Boundary value problems in the complex plane
30D45 Normal functions of one complex variable, normal families
30C62 Quasiconformal mappings in the complex plane

Keywords:

polynomially convex hull; analytic disc; holomorphic motion

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