Greenberg, Peter; Sergiescu, Vlad

An acyclic extension of the braid group. (English) Zbl 0736.20020

Comment. Math. Helv. 66, No. 1, 109-138 (1991).

The infinite Artin braid group was classically used as an algebraic setting to study the topological problem of classifying knots and links. Recently, there has been an explosive amount of activities arising from diverse directions (from the frontiers of theoretical physics, biology, etc.). For example, in the area of theoretical physics, one possible starting point to track down additional information is: [Braid group, Knot theory and Statistical mechanics, Eds. *C. N. Yang* and *M. L. Ge*, Adv. Ser. Math. Phys. 9, World Scientific (1989; Zbl 0716.00010)]. The curious readers should be aware that the rapid publication practiced in the physics world means that related references in these areas number in the thousands. There are new journals that specialize in knot theory and its applications (especially in physics). Many interesting mathematics papers are to be found in "hard core" physics journals (such as Nuclear Physics, Physical Reviews,...).

In the present work, the authors show that the infinite braid group B can be embedded as a normal subgroup of a group G with the integral homology of a trivial group so that G/B is the group of all PL-homeomorphisms of the interval $[0,1]$ with the property that the homeomorphisms are the identity near 0 and 1 and otherwise have slopes equal to integral powers of 2 (positive as well as negative) outside of a suitable finite number of points, all of them are in $Z[1/2]$ — i.e. 2- adic rational numbers.

Proposition 2.3 is called the "triangle rule" by the authors. It furnishes a "coordinate free" procedure for describing the braid group associated to discrete closed subsets of the plane. In the physics literature, a 'parametrized' version of this "triangle rule" is usually called the 'star-triangle equation' or the 'Yang-Baxter equation'.

The initial homological evidence supporting such an exact sequence of groups came from the path space fibration over the base space which is the loop space of the 3-sphere with fibre the second loop space of the 3- sphere. It was known that the Quillen plus construction applied to the classifying space of the infinite Artin braid group (as a discrete group) would yield a space of the same homotopy type as the second loop space of the 3-sphere. Thus the fibre has the correct integral homology. Earlier works of the authors led to a similar result relating the integral homology of the base loop space of the 3-sphere with the integral homology of the classifying space of the group F' which is isomorphic to G/B.

Thus, the task in the present work is to construct G. It involves the study of group actions at infinity on a tree as well as delooping techniques. The special role of the prime 2 appears to be connected with the fact that the infinite braid group is mapped surjectively onto the infinite symmetric group. The commutator quotient group of the latter is the cyclic group of order 2.

Lest the readers get carried away, the authors provided an example showing that path-fibrations whose base and fibres have the same integral homology as suitable groups do not always permit the construction of a corresponding acyclic group. The construction of the acyclic group exploits the idea of a group action at infinity on a tree and depends on earlier works, in particular, on a joint work of *E. Ghys* and *V. Sergiescu* [Comment. Math. Helv. 62, 185-239 (1987; Zbl 0647.58009)]. The last reference in the above paper is given in the form: [?] Notes manuscrites d'auteur inconnu decrivant des groupes simples, infinis, de presentation finie (dus a R. J. Thomson). L'un de ces groupes est G.

Such a reference poses an immediate challenge to a reviewer. A quick dash to the book shelf immediately identifies the reference as follows: *G. Higman* [Finitely presented infinite simple groups, Lect. Notes, Australian National University, Canberra (82 pages, c. 1980)]. The opening sentence of the preceding notes reads as follows: The aim of these lectures is to describe an infinite family of finitely presented infinite simple groups. R. Thomson discovered one of these groups...

The paper under review and its many ancestors as well as off-springs evidently hints at many interesting possible connections with physics. Perhaps the "simplest" and the "safest" statement to make is that the "configuration space" used by mathematicians coincides with the "physical space" of collections of "point particles"' in physics.

Reviewer: C.-H.Sah (Stony Brook)

MSC:

20F36 Braid groups; Artin groups
57Q45 Knots and links in high dimensions (PL-topology) (MSC2010)
58D05 Groups of diffeomorphisms and homeomorphisms as manifolds
55R35 Classifying spaces of groups and H-spaces in algebraic topology
20E22 Extensions, wreath products, and other compositions of groups
57R50 Differential topological aspects of diffeomorphisms
20J05 Homological methods in group theory

Keywords:

infinite Artin Braid Group; normal subgroup; integral homology; PL- homeomorphisms; triangle rule; star-triangle equation; Yang-Baxter equation; path space fibration; loop space; Quillen plus construction; classifying space; homotopy type; group actions; infinite symmetric group; acyclic group

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