Veys, W.

Relations between numerical data of an embedded resolution. (English) Zbl 0742.14009

Am. J. Math. 113, No. 4, 573-592 (1991).

The author considers the embedded resolution $h : X \to X_0$ of the singularities of a hypersurface Y in the affine space $X = \mathbb{A}^{n+1}$. Let $Y_i^{(r)}$, $i \in I$, be the strict transforms of the irreducible components of Y and $E_i^{(r)}$, $1 \leq i \leq r$, be the irreducible components of the exceptional divisor, then $\left(\bigcup_{i\in I} Y_i^{(r)}\right) \cup \left(\bigcup_{i=1}^r E_i^{(r)}\right)$ is a normal crossing divisor on X.

The numerical data (N_i, ν_i) are defined by: $h^{-1}(Y) = \sum_{i\in I} N_i Y_i^{(r)} + \sum_{i=1}^r N_i E_i^{(r)}$ and $K_X = h^{-1}(K_{X_0}) + \sum_{i\in I}(\nu_i - 1)Y_i^{(r)} + \sum_{i=1}^r (\nu_i - 1)E_i^{(r)}$. When Y is an irreducible plane curve there are some relations between these numbers and the author wants to generalize these results for any hypersurface $Y \subset \mathbb{A}^{n+1}$. He gets a relation for the canonical divisor on a divisor $E = E_j^{(r)}$, $1 \leq j \leq r$: Let E_i', $i \in T$, be the intersection $E_i^{(r)} \cap E$ or $Y_i^{(r)} \cap E$ of E with another component of $h^{-1}(Y)$, then $N_j K_E = \sum_{i\in T}((\nu_i - 1)N_j - \nu_j N_i)E_i'$ in $\mathrm{Pic}(E)$. — For a fixed $E_j^{(r)}$, he gets also some relations between the numerical data corresponding to the irreducible components $E_i^{(r)}$ which intersect $E_j^{(r)}$ and which appear "before $E_j^{(r)}$ in the resolution process". To get these relations he needs to look at the succession of blowing-up $g_i : X_{i+1} \to X_i$ with non-singular center D_i such that the map $X = X_r \to X_{r-1} \to \cdots \to X_0$ is the embedded resolution. Let h_j be the composed map $h_j : X_j \to X_0$. Let $E_j^{(r)}$ be the strict transform on X of the exceptional divisor $E = E_j^{(j)}$ of $g_{j-1} : X_j \to X_{j-1}$, i.e. $E = g_{j-1}^{-1}(D)$ with $D = D_{j-1}$, $\Pi = g_{j-1|_E}$, $k = \mathrm{codim}(D, X_{j-1})$. Let $E_i^{(r)}$, $i \in T$, $T \subset \{1, \dots, r\} \cup I$, be the irreducible components of $h^{-1}(Y)$ such that the $E_i' = E_i^{(r)} \cap E_j^{(r)}$ are the strict transforms in $E_j^{(r)}$ of the irreducible components of $E \cap (h_j^{-1}(Y) \backslash E)$, let $\alpha_i = (\nu_i - (\nu/N)N_i)$; then $\sum_{i\in T} d_i(\alpha_i - 1) + k = 0$, where d_i is the degree of the cycle $E_i'.F$ on the general fibre $F = \mathbb{P}^{k-1}$ of $\Pi : E \to D$. If $d_i = 0$ there exists a divisor B_i on D such that $E_i' = g_{j-1}^{-1}(B_i)$, and in $\mathrm{Pic}(D)$: $\sum_{i\in T, d_i\neq 0} \frac{1}{kd_i^{k-1}}(\alpha_i - 1)\Pi_*(E_i'^k) + \sum_{i\in T, d_i=0}(\alpha_i - 1)B_i = K_D$.

Reviewer: M.Vaquie (Paris)

MSC:

14E15 Global theory and resolution of singularities (algebro-geometric aspects)
14J17 Singularities of surfaces or higher-dimensional varieties

Cited in **6** Documents

Keywords:

embedded resolution of the singularities of a hypersurface; normal crossing divisor; blowing up; exceptional divisor

Full Text: DOI