

**Lascoux, Alain; Schützenberger, Marcel-Paul**

**Keys & standard bases.** (English) [Zbl 0815.20013](#)

Invariant theory and tableaux, Proc. Workshop, Minneapolis/MN (USA) 1988, IMA Vol. Math. Appl. 19, 125-144 (1990).

[For the entire collection see [Zbl 0694.00010](#).]

Let  $A$  be an ordered alphabet. A column is a word every letter in which is strictly less than the previous one. Every word  $w$  in the alphabet  $A$  can be uniquely written as a product of a minimal number of columns. Let  $L(w)$  be the least from them and let  $\|w\|$  be the sequence of their lengths. One column is greater than another if there exists a decreasing injection between the underlying sets of lengths. A tableau is a product of a decreasing sequence of columns. Every word  $w$  is equivalent to a unique tableau  $wR$  relative to the plastic congruence. A word is frank, if  $\|w\|$  is a permutation of  $\|wR\|$ . It appears that for every tableau  $t$  there exists another tableau  $K$  (right key) such that  $\|K\| = \|t\|$  and columns consist of  $L(f)$  for frank words  $f$  that are equal to  $t$ . A standard basis is a sum (in a free algebra) of all tableaux with the same right key. Those and dual (e.g. contertableau, left key) notions are investigated and applied efficiently e.g. for Schur functions, symmetric groups, Schubert varieties.

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**MSC:**

[20C30](#) Representations of finite symmetric groups  
[05E10](#) Combinatorial aspects of representation theory

Cited in **4** Reviews  
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**Keywords:**

Young tableaux; right key; ordered alphabet; number of columns; plastic congruence; frank words; standard basis; free algebra; left key; Schur functions; symmetric groups; Schubert varieties