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Borel's conjecture, Novikov's conjecture and the K-theoretic analogues. (English)

Zbl 0744.57018

Algebra, analysis, and geometry, Proc. Symp. in Honor of Chen-Jung Hsu and Kung-Sing Shih, Taipei/Taiwan 1988, 39-58 (1989).

[For the entire collection see Zbl 0723.00022.]

The article gives a survey over Borel's and Novikov's conjectures and the various variants and discusses proofs of these conjectures for special cases. The Borel conjecture says that two closed manifolds which are both Eilenberg MacLane spaces of type 1 are homeomorphic. Novikov's conjecture claims that the higher signature invariants of closed manifolds are homotopy invariants. Given a closed manifold M with total L-genus $L_*(M)$, a map $g : M \to K(\pi, 1)$ and $t \in H^s(K; Q)$, the higher signature of M associated with (K, g) is defined by $\langle L_*(M) \cup g^*(t), [M] \rangle$. These conjectures can be formulated in terms of surgery theory. There is a fibration $S^h(M) \to h(M, L(1)) \xrightarrow{\alpha} L(\pi_1(M))$. The strengthened Borel conjecture says that $S^h(M)$ is a point and α a homotopy equivalence whereas a reformulation of the Novikov conjecture claims that α is rationally a split morphism. There is also a K-theory analogue of this. The article explains the special case of a homotopy torus and deals with Novikov's conjecture respectively Borel's conjecture for manifolds admitting a Riemannian metric with non- positive respectively negative sectional curvature. The use of trace maps and cyclic homology is worked out. A sketch of the proof of the K- theoretic Novikov conjecture by Bökstedt, Madsen and the author using Waldhausen's A-theory is given.

Reviewer: W.Lück (Mainz)

MSC:

57R65	Surgery and handlebodies
57R67	Surgery obstructions, Wall groups
19D10	Algebraic K-theory of spaces
57-02	Research exposition (monographs, survey articles) pertaining to manifolds and cell complexes
57R20	Characteristic classes and numbers in differential topology
57R19	Algebraic topology on manifolds and differential topology

Keywords:

assembly map; manifolds admitting a Riemannian metric with non-positive sectional curvature; Novikov conjecture; Eilenberg MacLane spaces of type 1; higher signature invariants; homotopy invariants; L-genus; homotopy torus; trace maps; cyclic homology; A-theory; Borel conjecture; survey