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Topological classification of all integrable Hamiltonian differential equations of general type with two degrees of freedom. (English) Zbl 0753.58014

The geometry of Hamiltonian systems, Proc. Workshop, Berkeley/CA (USA) 1989, Math. Sci. Res. Inst. Publ. 22, 131-339 (1991).

[For the entire collection see Zbl 0733.00016.]

This paper is based on a series of lectures given by the author at the 1991 MSRI Hamiltonian system workshop. It contains a new theory of a topological classification of 2-dimensional integrable Hamiltonian systems (IHS). The main problems of this paper are: Consider a Hamiltonian system $\nu = \text{sgrad } H$ (skew gradient, H smooth) on a symplectic 4-manifold M^4 and let Q^3 be a compact isoenergy surface in M. Suppose ν is non-resonant and integrable (in the Liouville sense) on Q and let f be the second independent Bott integral (Morse function).

1. Enumeration problem: Does there exist an algorithm which enumerates all integrable Hamiltonian systems up to topological equivalence? (ν_1 and ν_2 are topological equivalent iff there exists a diffeomorphism which transforms the Liouville tori of the system ν_1 into the Liouville tori of the system ν_2 and preserves the orientation of the isoenergy surface).

2. Recognition problem: Does there exist an algorithm which solves the following problem: are two IHS topological equivalent or not?

3. Problem of algorithmical classification of all IHS in general position up to topological equivalence: Does there exist such an effective algorithm which can be implemented on the computer?

All 3 problems have affirmative answers and they are discussed in this paper. No proofs are given here but the reader is referred to the literature. The proof is based on the notion of skeletons and the complexity of IHS and the classification of isoenergy surfaces [the author, Math. USSR, Izv. 29, No. 3, 629-658 (1987); translation from Izv. Akad. Nauk SSSR, Ser. Mat. 50, No. 6, 1276-1307 (1986; Zbl 0619.58023)]. A diagrammer of graphs a la Feynman diagrams with associated letter-atoms and word-molecules is developed for coding the IHS and classifying the topological invariants $I(H, Q)$. A list of all IHS of low complexity is given and the real physical IHS are located in that table [*A. V. Bolsinov*, Russ. Math. Surv. 42, No. 6, 227-228 (1987); translation from Usp. Mat. Nauk 42, No. 6, 183-184 (1987; Zbl 0664.17006), Sov. Math., Dokl. 38, No. 1, 161-165 (1989); translation from Dokl. Akad. Nauk SSSR 301, No. 5, 1037-1040 (1988; Zbl 0676.58027), *A. V. Bolsinov*, the author, and *S. V. Matveev*, Russ. Math. Surv. 45, No. 2, 59-94 (1990); translation from Usp. Mat. Nauk 45, No. 2, 49-77 (1990; Zbl 0696.58019), *M. P. Kharlamov*, Sov. Math., Dokl. 28, 802-805 (1983); translation from Dokl. Akad. Nauk SSSR 273, No. 6, 1322-1325 (1983; Zbl 0561.58021) and *A. A. Oshemkov*, Russ. Math. Surv. 42, No. 6, 241-242 (1987); translation from Usp. Mat. Nauk 42, No. 6, 199-200 (1987; Zbl 0648.58016), Geometry, Differential Equ. and Mechanics, Moscow, 115-117 (1986), Baku internat. topolog. Conf., Abstracts, Part 2, Baku (1987), p. 230, Proc. Semin. Vector Tens. Analysis, Moscow, 122-132 (1988)]. The topological invariants are calculated for certain classical mechanical systems, e.g. 3-dimensional rigid bodies and Toda lattice.

The paper is organized into the following chapters: 1. The basic notations and the classification theorems. 2. The classification of isoenergy surfaces of integrable Hamiltonian systems. The remarkable class (H) of the three-dimensional manifolds. 3. Detailed descriptions of a new topological invariant for integrable Hamiltonian systems of differential equations. 4. Calculation of topological invariants of certain classical mechanical systems. 5. Class (H) of isoenergy surfaces of integrable Hamiltonian equations and hyperbolic geometry. 6. Multidimensional integrable Hamiltonian equations. Classifications of the surgery of Liouville tori in the neighborhood of the bifurcation diagram.

Reviewer: R.Schmid (Atlanta)

MSC:

37J35	Completely integrable finite-dimensional Hamiltonian systems, integration methods, integrability tests	Cited in **20** Documents
37K10	Completely integrable infinite-dimensional Hamiltonian and Lagrangian systems, integration methods, integrability tests, integrable hierarchies (KdV, KP, Toda, etc.)	
68Q25	Analysis of algorithms and problem complexity	

Keywords:

completely integrable systems; enumeration problem; recognition problem; problem of algorithmical classification; topological classification; effective algorithm; isoenergy surfaces