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Relations among some classes of functions in mathematical programming. (English)

Zbl 0742.49009
Mat. Metody Sots. Naukakh 22, 29-33 (1989).

For a measurable space $(T,\mathcal{T})$ and a separable metric space X the following classes of functions $f: T\times X\to[-\infty,+\infty)$ are considered: $F_1=\{f: f$ is $\mathcal{T}\otimes\mathcal{B}(X)$- measurable and upper semicontinuous in $x\}$, $F_2=\{f: f$ is a limit of a decreasing sequence of Carathéodory functions$\}$, $F_3=\{f: f$ is upper semicontinuous in x, and the set-valued map $t\to\{(x,r)\in X\times R:\ f(t,x)\geq r\}$ is measurable$\}$. These families arise in optimization and mathematical economics. Elements of F_3 are called normal integrands, cf. *R. T. Rockafellar* [in: Nonlin. Oper. Calc. Var., Summer Sch. Bruxelles 1975, Lect. Notes Math. 543, 157-207 (1976; Zbl 0374.49001)]. We study inclusions between these classes of functions; some of them were already known.

Always $F_3\subset F_2\subset F_1$. If $\mathcal{T}$ is closed under the Souslin operation and X is Souslin, then $F_1=F_2=F_3$. If T and X are Souslin spaces and $\mathcal{T}=\mathcal{B}(X)$, then $F_1=F_2$. If X is σ-compact, then $F_2=F_3$. We have examples that $F_1\neq F_2$ and $F_2\neq F_3$.

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MSC:

49J45	Methods involving semicontinuity and convergence; relaxation
54C30	Real-valued functions in general topology
26B99	Functions of several variables
49N99	Miscellaneous topics in calculus of variations and optimal control
90C99	Mathematical programming

Keywords:

Carathéodory functions; normal integrands