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Uniform estimates and blow-up behavior for solutions of $-\Delta u = V(x)e^u$ in two dimensions.

(English) [Zbl 0746.35006](#)

Commun. Partial Differ. Equations 16, No. 8-9, 1223-1253 (1991).

The problem under investigation is (*) $-\Delta u = fe^u$ in Ω , $u|_{\partial\Omega} = 0$ in a bounded domain Ω in R^2 , where $f \in L^p(\Omega)$ for some p in $1 < p \leq \infty$. If u is a solution of (*) with $e^u \in L^{p'}(\Omega)$, where p' denotes the conjugate exponent of p , one result states that $u \in L^\infty(\Omega)$. A detailed investigation is given of the delicate question of uniform estimates for a sequence $\{u_n\}$ satisfying $-\Delta u_n = f_n \exp u_n$ in Ω , $u_n|_{\partial\Omega} = 0$.

Theorem. If $f_n \geq 0$ in Ω and $\|f_n\|_p, \|\exp u_n\|_{p'}$ are uniformly bounded, where $\|\cdot\|_p$ denotes the norm in $L^p(\Omega)$, then $\{u_n\}$ is bounded in $L^\infty_{loc}(\Omega)$. Also conditions are found for which $\{u_n\}$ is bounded in $L^\infty(\Omega)$, and examples are constructed with $\|u_n\|_\infty \rightarrow \infty$ as $n \rightarrow \infty$, i.e., the uniform estimate does not hold up to the boundary of Ω in general.

Reviewer: Charles A. Swanson (Vancouver)

MSC:

35B45 A priori estimates in context of PDEs

35J65 Nonlinear boundary value problems for linear elliptic equations

Cited in **11** Reviews
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Keywords:

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