Sjöstrand, Johannes; Zworski, Maciej

Complex scaling and the distribution of scattering poles. (English) Zbl 0752.35046

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This important paper establishes sharp polynomial bounds on the number of scattering poles for a general class of compactly support self adjoint perturbations P of the Laplacian in $\mathbb{R}^n$, n odd.

Specifically if $N(r) = \#\{\lambda_j : |\lambda_j| \leq r, \quad \lambda_j \text{ is a scattering pole}\}$ and P satisfies certain structural assumptions, then $N(r) \leq C\Phi(Cr)$ for some constant C and $r \geq 1$. Here $\Phi : [1,\infty)$ is an increasing function such that $\Phi(t) \geq t^n$ and such that there exist $\tilde{C}$, $\delta > 0$ with the property $\Phi(\theta t) \leq \tilde{C}\theta^\delta \Phi(t)$ for all θ, t with $0 < \theta \leq 1$, $t \geq 1$, $\theta t \geq 1$.

This result covers all previously known cases. Furthermore the sharp polynomial bound on the number of scattering poles allows the application of the Tauberian theorem to obtain asymptotics of the scattering phase $s_{v,k}(\lambda)$ associated with the problem $-\Delta + V(x)$ on $O = \mathbb{R}^n \backslash K$ with Dirichlet boundary condition on ∂K, $V \in C_0^\infty(\overline{O})$.

Thus for scattering by a smooth compact obstacle K $S_{v,k}(\lambda) = C_n \operatorname{vol}(K)\lambda^n + O(\lambda^{n-1}), \lambda \to \infty$.

If the measure of the set of closed transversally reflected geodesics in $T^*(\mathbb{R}^n \backslash K)$ is zero then

$$S_{v,k}(\lambda) = C_n \operatorname{vol}(K)\lambda^n + C_n^1 \operatorname{vol}(\partial K)\lambda^{n-1} + O(\lambda^{n-1}).$$

These results generalise the work of Melrose who considered the corresponding Weyl law for the scattering phase in the obstacle case.

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MSC:

35P25	Scattering theory for PDEs
47A40	Scattering theory of linear operators
81U10	n-body potential quantum scattering theory

Keywords:

sharp polynomial bounds on the number of scattering poles; asymptotics of the scattering phase; shamp polynomial bounds on the number of scattering poles

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