Chui, C. K.; **Stöckler, J.**; **Ward, J. D.**

A Faber series approach to cardinal interpolation. (English) Zbl 0751.41002

Math. Comput. 58, No. 197, 255-273 (1992).

The paper is concerned with cardinal interpolation based on Faber polynomials. In §2 the authors give a brief introduction to Faber polynomials and in §3 some algorithmic constructions of Faber polynomials in regions G which are either sectors of disk or Moebius transform of the disk.

For $\varphi \in C_0(R^d)$ a compactly supported complex/valued function and $\Phi = (\varphi(j))_{j\in Z^d}$, one defines the symbol $\tilde\varphi$ by $\tilde\varphi(t) = \sum_{j\in\mathbb{Z}^d} \varphi(j) \centerdot \exp(ij \centerdot t)$, $t \in R^d$. Throughout the paper one supposes that $\tilde\varphi(t) \neq 0$ on R^d. The fundamental sequence $\Lambda = (\lambda_j)_{j\in Z^d}$ is defined by $\Lambda * \Phi = (\delta_{0j})$ (the Kronecker symbol) or equivalently $\tilde\Lambda = 1/\tilde\varphi$.

The cardinal interpolation operator is studied as the inverse of Schoenberg operator $S : \ell_2 \to \ell_2$, $a \to a*\Phi$ or, in symbol notation, $(Sa)^\sim = \tilde a\tilde\varphi$. The inverse T of S is given by $Tf = \Lambda * f$. In order to construct Tf numerically, the authors find approximations $\lambda^{(n)} \in \ell_1$ to Λ such that $\|\tilde\Lambda - \tilde\Lambda^{(n)}\|_\infty \to 0$, $n \to \infty$, namely $\Lambda^{(n)} = q_n^{(F)}(\Phi)$, where $q_n^{(F)}$ are the partial sums of the Faber series of $1/z$ in G. For symmetric φ, the rate of convergence to cardinal interpolant is superior to the one obtainable from the Neumann series, as given in *C. K. Chui*, [Multivariate splines, CBMS-NSF Reg. Conf. Ser. Appl. Math. 54, 189 p. (1988; Zbl 0687.41018)].

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MSC:

41A05 Interpolation in approximation theory
41A15 Spline approximation
41A63 Multidimensional problems (should also be assigned at least one other classification number from Section 41-XX)
65D05 Numerical interpolation

Keywords:

cardinal interpolation; Faber polynomials; Schoenberg operator

Full Text: DOI

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