Goodwillie, Thomas G.

Calculus. II: Analytic functors. (English) Zbl 0776.55008

K-Theory 5, No. 4, 295-332 (1992).

The paper is a sequel to "Calculus I" (Zbl 0741.57021), where derivatives of homotopy functors were introduced. Its contents is the development of the theory of ρ-analytic functors, i.e. homotopy functors F from spaces to (based) spaces or spectra satisfying the following property: Given an n-cube diagram $\mathcal{X}$ of spaces, which is an iterated homotopy pushout, and suppose the n maps from the initial vertex are all $(\rho+1)$-connected, then there is an integer q independent of n such that $F(\mathcal{X})$ behaves like an n-dimensional homotopy pullback diagram up to the dimension $(n-1)\rho-q$. A number of results are proved which imply the equivalence of ρ- analytic functors under additional connectivity assumptions once the equivalence of their differentials or derivatives is established.

To give a flavour of the main results we quote the "First Derivative Criterion": Let $F \to G$ be a natural transformation of ρ-analytic functors from spaces to spectra which induces homotopy equivalences of the differentials $D_X F(Y) \to D_X G(Y)$ for all X, Y. Then for any $(\rho+1)$-connected map $Y \to X$ the following diagram is a homotopy pullback

$$\begin{array}{ccc} F(Y) & \longrightarrow & G(Y) \\ \downarrow & & \downarrow \\ F(X) & \longrightarrow & G(X). \end{array}$$

As examples may serve Waldhausen's A-theory functor, which is 1-analytic, or the functor $X \mapsto \Omega^\infty\Sigma^\infty(\mathcal{T}\text{ op}(K,X)_+)$ with a finite CW-complex K, which is $\dim K$-analytic. A sketch of applications can be found in [the author, Proc. Int. Congr. Math., Kyoto/Japan 1990, Vol. I, 621-630 (1991; Zbl 0759.55011)].

The theory of analytic functors relies heavily on the homotopy theory of cubical diagrams. In the first three sections of the paper the author collects those results about cubes, n-dimensional homotopy pushouts and pullbacks and their homotopy theory spread over the literature which are needed for his theory.

For the convenience of the reader many of the results are reproved. Moreover, they are extended where extensions are necessary for the development of the theory. In the final two sections the author introduces ρ-analytic functors and proves the implications of analyticity mentioned above. For applications to A-theory, topological cyclic homology, and the free loop space functor Λ he shows in an appendix that the homotopy equivalence from the derivative $\partial_X\Omega^\infty\Sigma^\infty(\Lambda X_+)$ to $\Lambda\Sigma^\infty(\Omega X_+)$ established in "Calculus I" respects the S^1-action on both functors.

Reviewer: R.Vogt (Osnabrück)

MSC:

55P65 Homotopy functors in algebraic topology
19D10 Algebraic K-theory of spaces

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