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Simple algebras, base change, and the advanced theory of the trace formula. (English)

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Annals of Mathematics Studies, 120. Princeton, NY: Princeton University Press. xiii, 230 p. \$ 55.00/hbk; \$ 19.95/pbk (1989).

The subject of this book is an application of the theory of the invariant trace formula. Let E/F be a cyclic extension of number fields and $G = R_{E/F}GL(n)$. Then G has $GL(n)$ as an endoscopic group (in fact, a more general type of groups G is considered, comprising also the multiplicative group of central simple algebras over F ; they all have $GL(n)$ as an endoscopic group). The trace formula for G is compared with that for $GL(n)$. Essential is the correspondence $f \rightarrow f'$ between (Hecke) functions on G and functions on $GL(n)$ giving the transfer of orbital integrals. On the other side of the trace formulas we have then a correspondence between tempered representations of $G(F_v)$ and $GL(n, F_v)$. The comparison consists of an identification of terms in the geometric side of both trace formulas and the same for the representation theoretic side. Descent is an important tool here.

As an application global base change for $GL(n)$ is established for automorphic representations induced from cuspidal (and E/F of prime degree). Another part of the Langlands conjectures is the "induction" of automorphic representations from $GL(n, A_E)$ to $GL(n\ell, A_F)$, where $\ell = [E : F]$, which should correspond to induction of representations of the (still conjectural) Tannaka group. This correspondence of automorphic representations is derived here from the base change lifting.

Finally the strong Artin conjecture is proved for an extension of number fields with nilpotent Galois group, i.e. to any irreducible complex representation of degree n of the Galois group corresponds a cuspidal representation of $GL(n, A_F)$ with the same local L-factors at almost all places.

Reviewer: [J.G.M.Mars](#)

MSC:

- 11F70 Representation-theoretic methods; automorphic representations over local and global fields
- 11-02 Research exposition (monographs, survey articles) pertaining to number theory
- 11R39 Langlands-Weil conjectures, nonabelian class field theory
- 22E55 Representations of Lie and linear algebraic groups over global fields and adèle rings
- 12-02 Research exposition (monographs, survey articles) pertaining to field theory
- 22-02 Research exposition (monographs, survey articles) pertaining to topological groups
- 11R32 Galois theory

Cited in **10** Reviews
Cited in **179** Documents

Keywords:

Hecke functions; application; invariant trace formula; endoscopic group; orbital integrals; correspondence between tempered representations; Descent; global base change for $GL(n)$; automorphic representations; Langlands conjectures; induction of representations; Tannaka group; base change lifting; strong Artin conjecture; extension of number fields with nilpotent Galois group; cuspidal representation; local L-factors

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