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Fuzzy relation equations and their applications to knowledge engineering. Foreword by Lotfi A. Zadeh. (English) Zbl 0694.94025

Theory and Decision Library, Series D: System Theory, Knowledge Engineering and Problem Solving, 3. Dordrecht etc.: Kluwer Academic Publishers. xiii, 278 p. Dfl. 150.00; $ 79.00; £49.00 (1989).

The book covers the most useful and new results in solving fuzzy relation equations with applications to knowledge engineering. The lucid and logically well-organized exposition is divided into two parts. Firstly the resolution problem for fuzzy relation equations under a variety of compositions is thoroughly presented. The main topics are fuzzy relation equations in residuated lattices, the lower solutions of max-min fuzzy equations, the measure of fuzziness of solutions, decompositions, t- norms, approximate solutions. The second part provides the pleasure of seeing the connection between the theory developed in the earlier chapters and knowledge-based systems. Every chapter in the book is self- contained with own references and examples, completing the concepts.

The first chapter is introductory. In Chapter 2 the main problem of the book is described: solving fuzzy relation equations of the form

$$(1) \quad B = R \diamond A... \quad or \quad (2) \quad T = S \diamond Q...,$$

where A, B are fuzzy sets, R, S, T, Q are fuzzy relations and $\diamond$ stands for max-$\odot$, max-min, maxt-t, min-s, etc., compositions. For fuzzy relation equations on a residuated lattice L and $\diamond$ = max- $\odot$, the solvability of (2) is investigated. The study is focussed mainly on complete and dually complete Brouwerian lattices, determining the greatest (resp. smallest) solution.

In Chapter 3 the lower solutions of (1) and (2) are studied for $\diamond$ = max-min, if the referential sets are finite and L is linear. Different results are presented if L is either a complete Brouwerian or a complete completely distributive lattice.

Chapter 4 is devoted to the characterization of solutions of max-min fuzzy equations (on linear lattices with some additional properties) having the smallest and the greatest fuzzy entropy measure.

Chapter 5 deals with the greatest Boolean solution of max-min fuzzy equations defined on a complete bounded Brouwerian lattice L. If L is linear, minimal Boolean solutions are also characterized and a related optimization problem is discussed.

α-fuzzy relation equations are subject to Chapter 6. The main topic here is the fuzzy relation decomposition in the intersection (resp. union) of two sets. Various notions of transitivity and related properties of convergence of powers of the involved relations are studied.

Chapter 7 presents an algorithm which allows to calculate the square root of a fuzzy relation assigned on a linear bounded lattice. The concepts of triangular norm (t-norm) and conorm (s-norm) lead to natural generalizations of some fuzzy relation equations using max-t and min-s compositions. A detailed analysis of such equation is presented in Chapter 8 under the assumption that t is lower (resp. s is upper) semicontinuous. The results are extended to equations of complex structure.

A particular composition $\diamond$ is presented in Chapter 9, based on the evaluation of an equivalence (resp. difference) degree between a fuzzy set and a fuzzy relation. It leads to new operators and to corresponding relational equations, whose solution sets are entirely characterized.

Chapter 10 deals with inconsistent fuzzy relation equations. A solvability index is introduced as a numerical measure to what extent the fuzzy relation equation is solvable. Several techniques are proposed to answer the main question: how difficult is it to attain an enough good approximate solution and how to measure this property.

The next four chapters deal with applications of fuzzy relation equations in knowledge-based systems, for handling fuzziness in knowledge representation, for constructing the knowledge base, rule-based systems, inference algorithms. An expert fuzzy controller system for an industrial process is discussed.

The extensive bibliography in Chapter 15, concerning fuzzy relational equations and related topics, is

useful for all specialists, interested in the subject.

The book reflects the prolific authors research and the great reference erudition in the field.

Reviewer: K.Peeva

MSC:

94D05	Fuzzy sets and logic (in connection with information, communication, or circuits theory)
68T99	Artificial intelligence
68P20	Information storage and retrieval of data
94-02	Research exposition (monographs, survey articles) pertaining to information and communication theory

Keywords:

fuzzy relation equations; knowledge engineering; residuated lattices; lower solutions of max-min fuzzy equations; measure of fuzziness of solutions; decompositions; t-norms; approximate solutions; knowledge-based systems