Berrick, A. J.; Miller, C. F. III

Strongly torsion generated groups. (English) Zbl 0762.20017

Math. Proc. Camb. Philos. Soc. 111, No. 2, 219-229 (1992).

A group G is torsion generated (t.g.) if it is generated by its elements of finite order and a group G is strongly torsion generated (s.t.g.) if for every $n \geq 2$ there is an element $g \in G$ of order n such that the conjugates of g generate G. Examples of s.t.g. groups include the group A_∞ of even finitary permutations of a countable set, the subgroup $E(R)$ of the stable general linear group $GL(R)$ and the Steinberg groups $St(R)$, where R is an associative ring with 1.

Now s.t.g. groups were considered by *A. J. Berrick* [in J. Algebra 139, 190-194 (1991; Zbl 0745.20031)] where he proved that if A is an abelian group and $m \geq 2$, then there is a s.t.g. group G_m with $A \simeq H_m(G_m, \mathbb{Z})$ and $H_i(G_m, \mathbb{Z}) = 0$ $(1 \leq i < m)$. Here the authors, by using techniques of *G. Baumslag, E. Dyer* and *C. F. Miller* [Topology 22, 27-46 (1983; Zbl 0503.20018)] extend this result and show Theorem 1: Let $A_2, A_3, \dots$ be a sequence of abelian groups. Then there exists a s.t.g. group G such that $H_n(G, \mathbb{Z}) \simeq A_n$ for all $n \geq 2$. Moreover, if λ is an infinite cardinal and if each A_κ has cardinality $\leq \lambda$, then G can be chosen to be of cardinality λ and to have trivial centre. Then by using substantial results from homotopy theory they show Theorem 2: Let G be a group having only finitely many non-zero integral homology groups $H_n(G, \mathbb{Z})$. Then any complex linear representation $\phi : G \to GL_\kappa(\mathbb{C})$ is trivial on any finite subgroup of G. A consequence of Theorem 2 is that when a non-perfect group is generated by torsion elements its integral homology must be non-zero in infinitely many dimensions. Moreover, by Theorem 1 this result is best possible for torsion generated groups.

It was shown by *R. G. Swan* [in Proc. Am. Math. Soc. 11, 885-887 (1960; Zbl 0096.25302)] that the integral homology of a finite group must be non-zero in infinitely many dimensions. *H. Henn* obtained [in A note on the homology of locally finite groups (unpublished manuscript)] that if G is a locally finite group having only finitely many nonzero homology groups $H_n(G, \mathbb{Z})$, then G is acyclic. The authors here obtain more results and we mention Theorem 3. There exists a universal finitely presented acyclic group which is strongly torsion generated.

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MSC:

20J05 Homological methods in group theory
20F05 Generators, relations, and presentations of groups
20K40 Homological and categorical methods for abelian groups
20F50 Periodic groups; locally finite groups

Keywords:

elements of finite order; s.t.g. groups; integral homology groups; non-perfect groups; generated by torsion elements; torsion generated groups; locally finite groups; universal finitely presented acyclic group; strongly torsion generated

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