McIntosh, Richard J.

A generalization of a congruential property of Lucas. (English) Zbl 0755.11001

Am. Math. Mon. 99, No. 3, 231-238 (1992).

For a prime p and integers n,k let $n=n_0+n_1p+\cdots+n_rp^r$ $(0\le n_i\le p-1)$ and $k=k_0+k_1p+\cdots+k_rp^r$ $(0\le k_i\le p-1)$. If the functions $F(n)$ and $L(n,k)$ satisfy the congruences

$$F(n)\equiv F(n_0)F(n_1)\dots F(n_r)\mod p\quad\text{and}$$
$$L(n,k)\equiv L(n_0,k_0)L(n_1,k_1)\dots L(n_r,k_r)\mod p$$

for every prime p, then we say that F has the Lucas property (LP) and L has the double Lucas property (DLP). In 1878 Lucas proved that the binomial coefficient function $L(n,k)=\binom{n}{k}$ is a DLP function. The author presents various properties and connections on these functions. A typical result: If $L(n,k)$ is a DLP function, then $F(n)=\sum_{k=0}^{n}L(n,k)$ is an LP function.

Reviewer: P.Kiss (Eger)

MSC:

11A07	Congruences; primitive roots; residue systems
11B65	Binomial coefficients; factorials; q-identities
11A25	Arithmetic functions; related numbers; inversion formulas

Keywords:

congruences; Lucas property; double Lucas property; binomial coefficient

Full Text: DOI