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Higgs fibre bundles and local systems. (Fibrés de Higgs et systèmes locaux.) (French)

[Zbl 0762.14011](#)

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[For the entire collection see [Zbl 0742.00009](#).]

Donaldson (resp. Uhlenbeck and Yau) established a bijective correspondence between the set of equivalence classes of the irreducible and unitary representations of the fundamental group of a projective manifold (resp. compact Kähler manifold) X and the set of isomorphy classes of certain stable vector bundles. The crucial point of their works was to prove the existence of certain “good” hermitian metrics (Yang-Mills or Hermitian Einstein).

This article is an expository account of the works of C. Simpson who found an analogue for arbitrary linear representations. In the case where X is a curve, the main ingredient was the notion of Higgs bundle which was introduced and studied by *W. J. Hitchin* [Proc. Lond. Math. Soc., III. Ser. 55, 59-126 (1987; [Zbl 0634.53045](#))]. For algebraic vector bundles, equipped with some Higgs structure, and associated to certain representation $\pi \rightarrow GL(r, \mathbb{C})$, one has to select some good metrics. The author choose to do so via the notion of harmonic bundles. The bulb of the major works of Simpson, by using the results of Corlette and Donaldson, is to establish a categorical equivalence between those of linear representation of the fundamental group of a projective manifold, those of flat vector bundles and those of semistable Higgs bundles with trivial Chern classes. This amounts to find the existence of three coarse moduli spaces $M_B(r)$, $M_{DR}(r)$ and $M_{dol}(r)$ when the rank r of the bundle is fixed in advance.

The result of this article is to present a detailed account of the algebraic structure of those three spaces within the framework of projective manifolds, although Simpson’s work is carried on more general cases, namely Kähler manifolds.

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MSC:

[14F05](#) Sheaves, derived categories of sheaves, etc. (MSC2010)

[14F35](#) Homotopy theory and fundamental groups in algebraic geometry

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[Higgs bundle](#); [linear representation of the fundamental group](#); [coarse moduli spaces](#)

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