Björner, Anders

The homology and shellability of matroids and geometric lattices. (English) Zbl 0772.05027
Matroid applications, Encycl. Math. Appl. 40, 226-283 (1992).

[For the entire collection see Zbl 0742.00052.]

This survey studies the properties of three simplicial complexes that can be associated with a matroid M: (i) the matroid complex of independent sets of M; (ii) the broken circuit complex relative to an ordering of the ground set $E(M)$; and (iii) the order complex that consists of chains of flats in the geometric lattice associated with M. The broken circuit complex (ii) consists of those subsets of $E(M)$ which do not contain a broken circuit, the latter being a set that is obtained from a circuit by removing its least element.

The notion of shellability is used by the author as a framework for his discussion. A complex is shellable if all its maximal faces are equicardinal and those maximal faces can be ordered in a certain way that is favorable for induction arguments. All three complexes listed above are shellable. The author aims to give a "unified and concise, yet gentle, introduction" to these three complexes and the links between them. The survey assumes a minimum of prerequisites and, for its first half, takes an entirely combinatorial approach. All algebraic aspects of the subject are left for the second half. There is a short section presenting the relevant parts of simplicial homology that is designed to make the paper basically self-contained. The survey concludes with some historical remarks and with numerous exercises that supplement the results covered in the body of the paper.

Reviewer: J.G.Oxley (Baton Rouge)

MSC:

05B35	Combinatorial aspects of matroids and geometric lattices
06C10	Semimodular lattices, geometric lattices
18G99	Homological algebra in category theory, derived categories and functors

Keywords:

homology; sheallable complex; survey; matroid complex; broken circuit complex; geometric lattice; shellability