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Rarefactive solitary waves in two-phase fluid flow of compacting media. (English)

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Rarefactive solitary wave solutions of a third order nonlinear partial differential equation derived by *D. R. Scott* and *D. J. Stevenson* [Geophys. Res. Lett. 11, No. 11, 1161-1164 (1984)] to describe the one-dimensional migration of melt under the action of gravity through the Earth's mantle are investigated. The partial differential equation contains two parameters, n and m, which are the exponents in power laws relating, respectively, the permeability of the medium and the bulk and shear viscosities of the solid matrix to the voidage. It is proved that, for any value of m, rarefactive solitary wave solutions satisfying certain physically reasonable boundary conditions always exist if $n > 1$ but do not exist if $0 \leq n \leq 1$. It is also proved that the speed of the solitary wave is an increasing function of the amplitude of the wave. Six new exact rarefactive solitary wave solutions, four of which are expressed in terms of elementary functions and two in terms of elliptic integrals, are derived.

MSC:

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35Q51 Soliton equations
86A60 Geological problems

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