Boccardo, Lucio; Murat, François

Almost everywhere convergence of the gradients of solutions to elliptic and parabolic equations. (English) Zbl 0783.35020

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Let Ω be a bounded open set of $\mathbb{R}^N$, $1 < p, p' < \infty$, $1/p + 1/p' = 1$. Consider the nonlinear elliptic equations

$$-\operatorname{div} a(x, u_n, Du_n) = f_n + g_n \quad \text{in } \mathcal{D}'(\Omega) \tag{1}$$

where $a : \Omega \times \mathbb{R} \times \mathbb{R}^N \to \mathbb{R}^N$ is a Carathéodory function satisfying the classical Leray-Lions hypotheses. Assume that $u_n \rightharpoonup u$ weakly in $W^{1,p}(\Omega)$, strongly in $L^p_{\text{loc}}(\Omega)$ and a.e. in Ω, and $f_n \to f$ strongly in $W^{-1,p'}(\Omega)$. Moreover, assume that $g_n \in W^{-1,p'}(\Omega)$ is bounded in the space $\mathcal{M}(\Omega)$ of Radon measures.

In the present paper, the authors prove that $Du_n \to Du$ strongly in $\left(L^q(\Omega)\right)^N$ for any $q < p$. This implies that, for a suitable subsequence n', $Du_{n'} \to Du$ a.e. in Ω (cf. the title of the paper) and, moreover, that it is allowed to pass to the limit in (1) such that $-\operatorname{div} a(x, u, Du) = f + g$ in $\mathcal{D}'(\Omega)$.

Besides, under the stronger hypotheses $a(x, s, \zeta)\zeta \geq \alpha|\zeta|^p$ for some $\alpha > 0$ (a.e. $x \in \Omega$, and $s \in \mathbb{R}$, $\zeta \in \mathbb{R}^N$ arbitrary) and $g_n \rightharpoonup g$ weakly in $L^1(\Omega)$, they show that, for any fixed $k > 0$, the truncation T_k of u_n at height k satisfies $DT_k(u_n) \to DT_k(u)$ strongly in $\left(L^p_{\log}(\Omega)\right)^N$. Under suitably modified assumptions, corresponding results are obtained also in the parabolic case, i.e., when (1) is replaced by

$$\partial u_n/\partial t - \operatorname{div} a(x, t, u_n, Du_n) = f_n + g_n \quad \text{in } \mathcal{D}'\big(\Omega \times (0, T)\big)\ (T > 0 \text{ fixed}).$$

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MSC:

35J60 Nonlinear elliptic equations
35K55 Nonlinear parabolic equations
35B99 Qualitative properties of solutions to partial differential equations

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elliptic equations; parabolic equations; a.e. convergence of gradients

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