Lube, G.

An asymptotically fitted finite element method for convection dominated convection-diffusion-reaction problems. (English) Zbl 0772.65072

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The author presents convergence of an asymptotically fitted variant SDFEM-A of the finite element method of streamline-diffusion type (SDFEM) for singularly perturbed elliptic boundary value problems modelling convection-dominated convection-diffusion-reaction problems. The method is based on the observation that for $\varepsilon \leq C \cdot h$ and $\varepsilon \leq C \cdot h^{3/2}$, respectively, any unrefined mesh cannot resolve the downstream and characteristic boundary layers, respectively [cf. *C. Johnson*; *A. H. Schatz, L. B. Wahlbin*: Math. Comput. 49, 25-38 (1987; Zbl 0629.65111)].

The idea consists of replacing the sharp layers by smooth layers. As explained by *O. Axelsson* [I.M.A. J. Numer. Anal. 1, 329-345 (1981; Zbl 0508.76069)], this method can he viewed, in some sense as a limit case ($\varepsilon \ll h$) of using exponentially weighted functions.

Without perturbing the simple finite element shape structure and desirable linearization properties of SDFEM, the SDFEM-A allows for global error estimates in L_2-norm and sometimes in a weighted W_2'-norm which are uniformly valid with respect to ε. Such global results are not valid for SDFEM. As a result it is concluded that boundary layers are better approximated by the SDFEM-A.

Theoretical superiority of SDFEM-A is demonstrated through four numerical examples with the help of tables and graphs. Computed results establish that local oscillations of SDFEM-solutions in boundary layers are suppressed by SDFEM-A. Consequently SDFEM-A is in some situations an alternative to mesh refinement methods or exponentially fitted methods to resolve the layers.

The question of optimal local L_∞-estimates for SDFEM is still open. The paper by *C. Johnson* et al. (loc. cit.) concerning a modified streamline diffusion method SDFEM-C is a step forward to solve the problem.

The streamline diffusion schemes are now a common method for solving as well transport-dominated problems as more complicated convection- dominated flow problems, in particular incompressible and compressible Euler or Navier-Stokes equations.

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MSC:

65N30 Finite element, Rayleigh-Ritz and Galerkin methods for boundary value problems involving PDEs
65N50 Mesh generation, refinement, and adaptive methods for boundary value problems involving PDEs
65N15 Error bounds for boundary value problems involving PDEs
35J65 Nonlinear boundary value problems for linear elliptic equations

Keywords:

error estimates; finite element; streamline-diffusion; convection- dominated convection-diffusion-reaction problems; boundary layers; numerical examples; mesh refinement; exponentially fited method

Full Text: DOI

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