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Additive arithmetic functions and processes with independent increments. (English)

Zbl 0786.11050
Analytic and probabilistic methods in number theory. Proc. Int. Conf. in Honour of J. Kubilius, Palanga/Lith. 1991, New Trends Probab. Stat. 2, 211-221 (1992).

Given a "Kubilius-type" sequence f_n of additive functions $f_n : \{1,\dots,n\} \to G$, where G is a Hilbert space, the author is interested in conditions ensuring the weak convergence of the sequence of probability measures $\mathbb{P}_n(B) = \frac{1}{n}\cdot \#\{m \in \{1,\dots,n\};\ X_n(\cdot,m) \in B\}$, where $B \subset \{1,2,\dots,n\}$, and where

$$X_n(t,m) = \sum_{p \le y_n(t)} (f_n(p^{\alpha_p(m)}) - g_n(p))$$

with some centralizing elements $g_n(p) \in G$. The sequence f_n is of Kubilius type, if

$$\lim_{n\to\infty} \|f_n(p^\alpha)\| = 0, \qquad \lim_{n\to\infty} \max_{p\le n} \frac{\|f_n(p)\|}{p} = 0,$$

and if there is a sequence $r'_n \ge 1$, $\log r'_n = o(\log n)$ such that $\lim \sum_{r'_n<p\le n} \frac{1}{p}\|f_n(p)\|^2 = 0$.

Theorem 1 describes the set of limit points of $\{\mathbb{P}_n;\ n \ge 1\}$.

Theorem 2 gives necessary and sufficient conditions for the weak convergence of $\mathbb{P}_n$:

(1) $\sup_{0\le t\le 1} \|m_n(t) - m(t)\| \to 0$, as $n \to \infty$, where $m_n(t) = -\sum_{p\le y_n(t)} \frac{\|f_n(p)\|^2}{p(1+\|f_n(p)\|^2)} f_n(p)$,

(2) the sequence $\{T_n(1);\ n \ge 1\}$ is compact, where $(T_n(t)\cdot x, y) = \sum_{\substack{p\le y_n(t)\\ \|f_n(p)\|\le 1}} \frac{1}{2}(f_n(p),x)\cdot(f_n(p),y)$, and two further (complicated) conditions.

Theorem 3 deals with the (weak) convergence of distribution functions.

[For the entire collection see Zbl 0754.00023.]

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MSC:

11K65 Arithmetic functions in probabilistic number theory

Keywords:

Hilbert space-valued additive functions; weak convergence of probability measures; Skorokhod topology; arithmetical modelling of stochastic processes with independent increments; Kubilius-type sequences; Kubilius fundamental lemma; independent random variables; functional limit theorems; compact sequences of operators; (weak) convergence of distribution functions