

**Scharlemann, Martin**

**Unlinking via simultaneous crossing changes.** (English) Zbl 0785.57003  
Trans. Am. Math. Soc. 336, No. 2, 855-868 (1993).

The following problem is considered: given two distinct crossings of a knot or link projection, under what conditions can one obtain the unlink by changing both crossings simultaneously? More precisely, consider two “crossing disks”  $D_1$  and  $D_2$  for a link  $L$ , i.e. disks intersecting  $L$  in exactly two points, and twist the link  $t_1$  [resp.  $t_2$ ] times as it passes through  $D_1$  [resp.  $D_2$ ] (adding  $2t_1 + 2t_2$  new crossings to the projection of the link). Denote the new link by  $L(t_1, t_2)$ . The main result of the paper states that if the disks are essential and do not bound an annulus disjoint from  $L$ , there exists a pair  $(s_1, s_2)$  of integers with the following property: whenever  $L(t_1, t_2)$  is the unlink, either  $t_1 = s_1$  or  $t_2 = s_2$ . This is a consequence of a more general result giving an analogous statement for pairs  $(t_1, t_2)$  reducing the minimal Thurston norm among all spanning surfaces of the link disjoint from the boundary of the two disks.

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**MSC:**

57M25 Knots and links in the 3-sphere (MSC2010)  
57M40 Characterizations of the Euclidean 3-space and the 3-sphere (MSC2010)

Cited in 4 Documents

**Keywords:**

[changing crossings simultaneously](#); [Thurston norm](#); [spanning surfaces](#); [link](#)

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**References:**

- [1] John W. Morgan and Hyman Bass, The Smith conjecture, Pure and Applied Mathematics, vol. 112, Academic Press, Inc., Orlando, FL, 1984. Papers presented at the symposium held at Columbia University, New York, 1979. · [Zbl 0599.57001](#)
- [2] Steven Bleiler and Martin Scharlemann, Tangles, property  $\lambda$ , and a problem of J. Martin, Math. Ann. 273 (1986), no. 2, 215 – 225. · [Zbl 0563.57002](#) · [doi:10.1007/BF01451402](https://doi.org/10.1007/BF01451402) · [doi.org](https://doi.org/)
- [3] F. Bonahon, Involutions et fibrés de Seifert dans les variétés de dimension  $\leq 3$ , Thèse de 3e cycle, Orsay, 1979.
- [4] Gerhard Burde and Heiner Zieschang, Knots, De Gruyter Studies in Mathematics, vol. 5, Walter de Gruyter & Co., Berlin, 1985. · [Zbl 0568.57001](#)
- [5] David Gabai, Foliations and the topology of 3-manifolds, J. Differential Geom. 18 (1983), no. 3, 445 – 503. · [Zbl 0533.57013](#)
- [6] C. McA. Gordon and J. Luecke, Reducible manifolds and Dehn surgery, Topology 35 (1996), no. 2, 385 – 409. · [Zbl 0859.57016](#) · [doi:10.1016/0040-9383\(95\)00016-X](https://doi.org/10.1016/0040-9383(95)00016-X) · [doi.org](https://doi.org/)
- [7] W. B. Raymond Lickorish, The unknotting number of a classical knot, Combinatorial methods in topology and algebraic geometry (Rochester, N.Y., 1982) Contemp. Math., vol. 44, Amer. Math. Soc., Providence, RI, 1985, pp. 117 – 121. · [doi:10.1090/conm/044/813107](https://doi.org/10.1090/conm/044/813107) · [doi.org](https://doi.org/)
- [8] José M. Montesinos, Surgery on links and double branched covers of  $S^3$ , Knots, groups, and 3-manifolds (Papers dedicated to the memory of R. H. Fox), Princeton Univ. Press, Princeton, N.J., 1975, pp. 227 – 259. Ann. of Math. Studies, No. 84.
- [9] Martin Scharlemann, Sutured manifolds and generalized Thurston norms, J. Differential Geom. 29 (1989), no. 3, 557 – 614. · [Zbl 0673.57015](#)
- [10] Martin Scharlemann, Handlebody complements in the 3-sphere: a remark on a theorem of Fox, Proc. Amer. Math. Soc. 115 (1992), no. 4, 1115 – 1117. · [Zbl 0759.57012](#) ·
- [11] Martin Scharlemann, Producing reducible 3-manifolds by surgery on a knot, Topology 29 (1990), no. 4, 481 – 500. · [Zbl 0727.57015](#) · [doi:10.1016/0040-9383\(90\)90017-E](https://doi.org/10.1016/0040-9383(90)90017-E) · [doi.org](https://doi.org/)
- [12] Martin Scharlemann and Abigail Thompson, Link genus and the Conway moves, Comment. Math. Helv. 64 (1989), no. 4, 527 – 535. · [Zbl 0693.57004](#) · [doi:10.1007/BF02564693](https://doi.org/10.1007/BF02564693) · [doi.org](https://doi.org/)
- [13] William P. Thurston, A norm for the homology of 3-manifolds, Mem. Amer. Math. Soc. 59 (1986), no. 339, i – vi and 99 – 130. · [Zbl 0585.57006](#)

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