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Singularities and groups in bifurcation theory. Volume II. (English) Zbl 0691.58003

Applied Mathematical Sciences, 69. New York etc.: Springer-Verlag. xvi, 533 p. DM 134.00 (1988).

The aim of volume I (1985; Zbl 0607.35004) of this work was to study applications of singularity theory to bifurcation, showing that unfolding degenerate singularities illustrates complicated bifurcation diagrams. A very important new ingredient, namely symmetry, both in the equations and in their solutions, is introduced in volume II.

Starting from the symmetry group G of an ordinary differential equation (ODE), the isotropy subgroup of a steady-state solution x_0 is defined as $\Sigma = \{g \in G : gx_0 = x_0\}$. One of the main problems in this book is "generically, for which isotropy subgroups Σ should we expect to find bifurcating branches of steady-states having Σ as their group of symmetries" (p. 8). The fact that G- invariant solutions break into a branch of solutions with symmetry group Σ is named spontaneous symmetry-breaking. It is remarked that bifurcation problems become more complicated when symmetry is involved because this usually forces eigenvalues to have high (≥ 2) multiplicity, which implies, via the Lyapunov-Schmidt reduction, bifurcation equations with several variables. But, in turn, the very same symmetry complicating bifurcation provides techniques to deal with the corresponding difficulties. Multiplicity of eigenvalues is closely related with irreducible group actions, because all compact Lie groups, except 1 and $Z_2 = \{\pm 1\}$, have irreducible actions on vector spaces of dimension > 1.

An equivariant singularity theorem is developed in this volume in order to adapt the framework in vol. I to the presence of symmetry. Another main topic is mode interaction, i.e., complications created by several eigenvalues crossing simultaneously the imaginary axis. As a rule, the study of Hopf bifurcation (for ODEs) plays a very important role all along the book. Now we shall give a general idea of the contents of vol. II. More comments will follow at the end.

Chapter 12 gives an introduction to Lie groups (in $\mathbb{R}^n$) and the main ideas concerning actions and group representations, irreducible representations and invariant theory. Symmetry-breaking in steady-state bifurcation is studied in Chapter 13, where the geometry of group actions, with the isotropy group Σ and the fixed-point subspace $\mathrm{Fix}(\Sigma)$, is described. The main result is the equivariant branching theorem, with applications to orbital stability.

Chapters 14 and 15 cover the algebraic part, namely Singularity theory, giving equivariant versions of Chapters 2 and 3 in vol. I. The first deals with equivariant singularity theory and the recognition problem, and the proof of the equivariant universal unfolding theorem is included in the second. Results are applied to the spherical Bénard problem.

Chapter 16 includes a general theory of equivariant Hopf bifurcation in the nondegenerate case. This is applied in the following chapter to the case of circular symmetry, where nonlinear degeneracies are allowed. Some difficult applications are in Chapter 18, in particular a model for a ring of coupled cells. The last two chapters are devoted to mode interactions, and it is shown that degeneracies arising from multiple purely imaginary eigenvalues provide very rich bifurcation phenomena.

This second volume has the same nice features of its companion. Examples and Case Studies are developed with taste and great care. From the beginning, the deformation of an elastic cube and the oscillations of a hosepipe are used as a motivation for the basic notions in the theory. Later, Bénard and Taylor's problems in Fluid Mechanics provide excellent tests for the tools developed at length in the text. Heuristics plays a very important role. We just single out, between many other possibilities, the treatment of model-dependent and model-independent properties, spontaneous symmetry-breaking (p. 68), comments on genericity (p. 81) and Birkhoff's normal forms (p. 310), and the explanation and use of schematic bifurcation diagrams (Chap. 13). The treatment of the Taylor-Couette system, at the end of the book, is a beautiful example, combining the goals (p. 486), conjectures (p. 494), physical experiments, and the employ of the different mathematical techniques. Tables and diagrams are exhaustive and illustrative. The book is as readable as possible with such a difficult and technical subject.

Reviewer: J.Hernandez

MSC:

58-02 Research exposition (monographs, survey articles) pertaining to global analysis
58C25 Differentiable maps on manifolds
58K99 Theory of singularities and catastrophe theory
34C25 Periodic solutions to ordinary differential equations
35B32 Bifurcations in context of PDEs
20C32 Representations of infinite symmetric groups

Keywords:

isotropy group; irreducible representation; singularity theory; bifurcation; symmetry; spontaneous symmetry-breaking; equivariant singularity theory; equivariant Hopf bifurcation