Golubov, B. I.; Efimov, A. V.; Skvortsov, V. A.

Walsh series and transforms. Theory and applications. (Ryady i preobrazovaniya Uolsha. Teoriya i primeneniya.) (Russian) Zbl 0692.42009

Moskva: Nauka. 344 p. R. 3.30 (1987).

Considerable part of the book is devoted to systematical study of Walsh system. Here is the list of main topics: Lebesgue constants of Walsh system; absolutely and uniformly convergent Walsh-Fourier series; uniqueness theorems; summability of Walsh-Fourier series by Cesàro and other regular methods of summation; Walsh-Fourier series in L_p $(1 \le p < \infty)$; Walsh-Fourier series with coefficients tending monotonically to zero; Rademacher and other lacunary subsystems of Walsh system; central limit theorem for lacunary subsystems; existence of L^1-functions with everywhere divergent Walsh-Fourier series; almost everywhere convergence of Walsh-Fourier series of L^2-functions; best approximation of functions by Walsh and Haar polynomials.

The rest of the book is devoted to multiplicative systems, which are generalizations of Walsh system, and to multiplicative transforms. Here are their definitions. Let $P = (p_1, p_2, ..., p_j, ...)$; p_j are integers, $p_j \ge 2$. Let $m_0 = 1$; $m_j = \prod_{s=1}^{j} p_s$. For every natural number n we have a unique representation $n = \sum_{j=1}^{k} a_j m_{j-1}$, where a_j are integers, $0 \le a_j \le p_j - 1$. For every $x \in [0, 1)$ we have a representation $x = \sum_{j=1}^{\infty} (x_j/m_j)$, where x_j are integers, $0 \le x_j \le p_j - 1$. Such representation is unique if we exclude representations in which all x_j except a finite number of them are equal $p_j - 1$. Multiplicative system corresponding to P is the system $\chi_n(x) = \exp(2\pi i \sum_{j=1}^{k} (a_j x_j/p_j))$. Let $\mathcal{P} = (..., p_{-j}, ..., p_{-1}, p_1, ..., p_j, ...)$, where p_j are integers, $p_j \ge 2$. For every $x, y \in [0, \infty)$ we can find representations $x = \sum_{j=1}^{k} x_{-j} m_{-j+1} + \sum_{j=1}^{\infty} (x_j/m_j)$; $y = \sum_{j=1}^{m} y_{-j} m_{j-1} + \sum_{j=1}^{\infty} (y_j/m_{-j})$, where x_j, y_j are integers; $0 \le x_j \le p_j - 1$; $0 \le y_j \le p_{-j} - 1$. These representations are unique if we make above mentioned exclusion. The multiplicative transform corresponding to $\mathcal{P}$ is defined in the following way. Let $f \in L^1[0, \infty)$. Then

$$(1) \quad \hat{f}(y) = \int_0^{\infty} f(x) \exp(2\pi i (\sum_{j=1}^{m} (x_j y_{-j}/p_j) + \sum_{j=1}^{k} (x_{-j} y_j/p_{-j}))) dx.$$

The book contains results on representation of functions using their multiplicative transforms. An interest to multiplicative transforms and their active use in applications are based upon the properties of their discretizations, i.e. transforms which is obtained if the integral in (1) is replaced by certain integral sums.

The book contains descriptions of the discretizations of the multiplicative transforms and investigation of the obtained discrete transforms. Here we find also descriptions of several applications of multiplicative systems and transforms.

Reviewer: M.Ostrovskij

MSC:

42C10 Fourier series in special orthogonal functions (Legendre polynomials, Walsh functions, etc.)
42C05 Orthogonal functions and polynomials, general theory of nontrigonometric harmonic analysis
42-01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to harmonic analysis on Euclidean spaces
40G05 Cesàro, Euler, Nörlund and Hausdorff methods

Keywords:
Walsh system; Lebesgue constants; Walsh-Fourier series; multiplicative systems; applications