Selberg, Atle

Old and new conjectures and results about a class of Dirichlet series. (English) Zbl 0787.11037

Bombieri, E. (ed.) et al., Proceedings of the Amalfi conference on analytic number theory, held at Maiori, Amalfi, Italy, from 25 to 29 September, 1989. Salerno: Universitá di Salerno, 367-385 (1992).

Let $F(s) = \sum a_n n^{-s}$ be a Dirichlet series with $a_n \ll n^\varepsilon$ for any $\varepsilon > 0$. Assume that there is an analytic continuation to an entire function, except possibly for a pole at $s = 1$, and suppose there is a functional equation of the usual type. Suppose further that $\log F(s)$ also has a Dirichlet series $\sum b_n n^{-s}$ with b_n supported on the prime powers, and satisfying $b_n \ll n^\vartheta$ for some $\vartheta < \frac{1}{2}$. Various conjectures on such functions are presented, which can be viewed as a very low-brow alternative to the Langlands philosophy.

For example it is conjectured that if $F_1(s)$ and $F_2(s)$ cannot be factorized into other functions of the same type then

$$\sum_{p\le x} a_{1p}\overline{a_{2p}}/p = \delta \log\log x + O(1),$$

where $\delta = 1$ or 0 depending on whether $F_1 = F_2$ or not.

Subject to certain hypotheses on the zeros of $F(s)$, the value distribution of $\log F(\sigma + it)$ for fixed σ near $\frac{1}{2}$ is found, which permits an investigation of the "a-points" of $F(s)$ (i.e. the zeros of $F(s) - a$). Finally similar questions for linear combinations $\sum_1^n c_i F_i(s)$ are considered.

For the entire collection see [Zbl 0772.00021].

Reviewer: D.R.Heath-Brown (Oxford)

MSC:

11M41	Other Dirichlet series and zeta functions
11R39	Langlands-Weil conjectures, nonabelian class field theory

Keywords:

Euler product; a-points; Dirichlet series; functional equation; alternative to Langlands philosophy; zeros; value distribution