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Strong approximation for set-indexed partial sum processes via KMT constructions. I. (English) Zbl 0776.60045

Ann. Probab. 21, No. 2, 759-790 (1993).

Summary: Let $(X_i)_{i\in\mathbb{Z}_+^d}$ be an array of independent identically distributed zero-mean random vectors with values in $\mathbb{R}^k$. When $E(|X_1|^r)<+\infty$, for some $r>2$, we obtain the strong approximation of the partial sum process $(\sum_{i\in\nu S}X_i : S\in\mathcal{S})$ by a Gaussian partial sum process $(\sum_{i\in\nu S}Y_i : S\in\mathcal{S})$, uniformly over all sets in a certain Vapnik-Chervonenkis class $\mathcal{S}$ of subsets of $[0,1]^d$. The most striking result is that both an array $(X_i)_{i\in\mathbb{Z}_+^d}$ of i.i.d. random vectors and an array $(Y_i)_{i\in\mathbb{Z}_+^d}$ of independent $N(0,\mathrm{Var}X_1)$-distributed random vectors may be constructed in such a way that, up to a power of $\log\nu$,

$$\sup_{S\in\mathcal{S}}\left|\sum_{i\in\nu S}(X_i-Y_i)\right| = O(\nu^{(d-1)/2}\vee\nu^{d/r}) \text{ a.s.},$$

for any Vapnik-Chervonenkis class $\mathcal{S}$ fulfilling the uniform Minkowsky condition.

From a paper of *J. Beck* [Z. Wahrscheinlichkeitstheorie Verw. Geb. 70, 289-306 (1985; Zbl 0554.60037)], it is straightforward to prove that such a result cannot be improved, when $\mathcal{S}$ is the class of Euclidean balls.

MSC:

60F17	Functional limit theorems; invariance principles
62G99	Nonparametric inference

Keywords:

central limit theorem; set-indexed process; invariance principle; metric entropy; random measure; strong approximation; Vapnik-Chervonenkis class

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