**Friedman, Charles N.**
**Sums of divisors and Egyptian fractions.** (English) Zbl 0781.11015
J. Number Theory 44, No. 3, 328-339 (1993).

The author discusses the presentation of rational numbers as a sum of Egyptian fractions, i.e. fractions of the form $1/X_i$, $X_i$ integers $> 1$, and related problems. A number $n$ is called abundant, if the sum of all positive divisors of $n$ is $\geq 2n$. If $\mathbf{p} = (p_1, p_1, \dots, p_k)$ is a vector of different primes and $\mathbf{a} = (a_1, a_2, \dots, a_k)$ is a vector of nonnegative integers, then we write $\mathbf{p}^{\mathbf{a}} = p_1^{a_1} p_2^{a_2} \cdots p_k^{a_k}$ and the vector $\mathbf{p}$ is called abundant, if some number of the form $\mathbf{p}^{\mathbf{a}}$ is abundant. The author shows that a necessary and sufficient condition for $\mathbf{p}$ to be abundant is: $\prod_i p_i/(p_i - 1) \geq 2$.

He proves the following theorem. Suppose that $\mathbf{p} = (p_1, p_2, \dots, p_k)$ is a fixed vector of successive primes with $p_k < p_1^r < 2p_k$ for some integer $r$ and $\mathbf{p}$ is abundant. Suppose that for each integer $\xi$ with $1 < \xi < p_1$ an equation of the form $\xi \mathbf{p}^{\mathbf{b}} = \mathbf{p}_1^{\mathbf{c}_1} + \cdots + \mathbf{p}_j^{\mathbf{c}_j}$ holds, where $\mathbf{p}^{\mathbf{b}} > 1$ and $\mathbf{c}_i$ are distinct. Then every rational positive number $X$ of the form $A/\mathbf{p}^{\mathbf{a}}$ has an Egyption fraction representation $X = 1/X_1 + \cdots + 1/X_n$ where $X_i$ distinct, of the form $\mathbf{p}_i^{\mathbf{a}}$. As an example he shows $\mathbf{p} = (3, 5, 7)$ and $1 = 1/3 + 1/5 + 1/7 + 1/9 + 1/15 + 1/21 + 1/27 + 1/35 + 1/45 + 1/105 + 1/945$.

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**MSC:**

11D68 Rational numbers as sums of fractions
11A25 Arithmetic functions; related numbers; inversion formulas



**Keywords:**

semiperfect numbers; weird numbers; abundant numbers; rational numbers; sum of Egyptian fractions

**Full Text:** DOI