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Nonhomogeneous Riemannian 3-manifolds with distinct constant Ricci eigenvalues. (English)

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A Riemannian manifold (M,g) is said to be curvature homogeneous if, for each pair of points p and q, there exists a linear isometry F between the tangent spaces T_pM and T_qM such that $F^*(R_q)=R_p$ where R denotes the Riemannian curvature tensor. Moreover, if $(\overline{M},\overline{g})$ is a homogeneous Riemannian manifold and o a fixed point of $\overline{M}$, then (M,g) has the same curvature tensor as $(\overline{M},\overline{g})$ if, for each $p\in M$, there exists a (linear) isometry $F:T_pM\to T_oM$ such that $F^*(\overline{R}_o)+R_p$. In this case, $(\overline{M},\overline{g})$ is called a homogeneous model space of (M,g). Obviously, any (locally) homogeneous space is curvature homogeneous but many examples which are not locally homogeneous are now known.

For three-dimensional manifolds the curvature homogeneity is equivalent to the constancy of the Ricci eigenvalues ρ_1,ρ_2,ρ_3. In [Nagoya Math. J. 132, 1-36 (1993; Zbl 0788.53038)] the author made a detailed study of the case $\rho_1=\rho_2\neq\rho_3$ and obtained examples of curvature homogeneous spaces without a homogeneous model. The existence of curvature homogeneous ones with prescribed arbitrary and "distinct" Ricci eigenvalues has been proved by A. Spiro and F. Tricerri but they did not provide explicit examples. On the other hand K. Yamato found complete examples in this class but they all have a homogeneous model. In this paper the author extends the class of Yamato examples and provides noncomplete curvature homogeneous ones without any homogeneous model. His method is based on J. Milnor's classification of three-dimensional Lie groups equipped with a left invariant metric.

Reviewer: L.Vanhecke (Heverlee)

MSC:

53C20 Global Riemannian geometry, including pinching
53C30 Differential geometry of homogeneous manifolds

Keywords:

curvature homogeneous; homogeneous model space; Ricci eigenvalues

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