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Strong approximation for set-indexed partial-sum processes, via KMT constructions. II.
(English) Zbl 0779.60030
Ann. Probab. 21, No. 3, 1706-1727 (1993).

Summary: [For part I see ibid. 21, No. 2, 759-790 (1993; Zbl 0776.60045).]

Let $(X_i)_{i\in\mathbb{Z}_+^d}$ be an array of zero-mean independent identically distributed random vectors with values in $\mathbb{R}^k$ with finite variance, and let $\mathcal{S}$ be a class of Borel subsets of $[0,1]^d$. If, for the usual metric, $\mathcal{S}$ is totally bounded and has a convergent entropy integral, we obtain a strong invariance principle for an appropriately smoothed version of the partial-sum process $\{\sum_{i\in\nu S} X_i : S\in\mathcal{S}\}$ with an error term depending only on $\mathcal{S}$ and on the tail distribution of X_1. In particular, when $\mathcal{S}$ is the class of subsets of $[0,1]^d$ with α-differentiable boundaries introduced by *R. Dudley* [J. Approximation Theory 10, 227-236 (1974; Zbl 0275.41011)], we prove that our result is optimal.

MSC:

60F17 Functional limit theorems; invariance principles
62G99 Nonparametric inference

Keywords:

central limit theorem; set-indexed process; invariance principle; metric entropy with inclusion; multivariate empirical processes; strong invariance principle

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