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Measure-valued Markov processes. (English) Zbl 0799.60080

Hennequin, P. L. (ed.), Ecole d'Eté de probabilités de Saint-Flour XXI - 1991, du 18 Août au 4 Septembre, 1991. Berlin: Springer-Verlag. Lect. Notes Math. 1541, 1-260 (1993).

This is an introduction and survey to the rapidly growing field of measure-valued stochastic processes. They have their origin in branching processes, population genetics models, interacting particle systems and stochastic partial differential equations. The main emphasis is given to two building blocks of measure-valued processes: superprocesses and Fleming-Viot processes.

Measure-valued processes play an important role in providing a rich class of mathematically tractable models. There are several reasons for this. First of all, there are three complementary mathematical structures which can be exploited: (i) Duality leads in the case of measure-valued branching processes to the log-Laplace nonlinear evolution equation which links the subject with (deterministic) analysis, or in the case of Fleming-Viot processes to a function-valued dual stochastic process which is more tractable in some sense. (ii) The family structure involves the family relationships and past history of the population, is tractable with probabilistic tools, and plays an important role in studying both the sample path behavior and long-time behavior. (iii) The martingale characterization uses powerful tools of stochastic analysis, yields a modified approach to both basic classes of processes and opens perspectives to wider classes of measure-valued processes, in particular, to processes with interaction. Secondly, these processes exhibit interesting dimension dependent local spatial structure and interesting long-time scale behavior, and may serve as test case for more complex models.

The objective of these notes is to provide an introduction to these different aspects of measure-valued processes with an emphasis on interrelations and some survey on approaches currently under development. Main attention is paid to outline main lines of development rather than attempting a systematic detailed exposition. Often simplifications are imposed to keep the exposition as simple as possible.

Here is a brief description of the 12 chapters of the notes: In the introductory chapter the roots of the theory of measure-valued Markov processes and the major topics of the notes are outlined. A natural approach to measure-valued processes and a useful interpretation of them is to start with approximating large stochastic systems of particles with small mass, and scale them appropriately. This intuitive approach is used in Chapter 2 to introduce two basic classes of measure-valued processes by rather elementary methods, but which may serve as a guiding principle also for later chapters. At the same time some basic notions are introduced, particularly, generators of measure-valued processes, Moran particle processes, moment equations, and martingale problems. In Chapter 3 some basic material is collected as random measures on Polish spaces, canonical measure-valued processes, Markov processes and weak convergence of random measures and measure-valued processes, to make the notes self- contained to some extent. The construction of different general versions of measure-valued branching processes via a particle approach is the topic of Chapter 4. A central role is played here by the log-Laplace evolution equation. Chapter 5 is devoted to the martingale problem approach to probability-measure-valued processes, particle approximations, duality relations, a discussion of a number of variations of the basic Fleming-Viot model arising in population genetics, and some other classes of measure-valued processes with function-valued dual. The martingale problem formulation of several versions of measure-valued branching is the subject of Chapter 6, which also reviews some basic facts from stochastic calculus. This will be the base for the development of a stochastic calculus for superprocesses in the next chapter: martingale measures, stochastic integrals and, as a key point, the Cameron-Martin-Girsanov formula for measure-valued diffusions. Chapter 8 deals with structure properties of branching and sampling martingale problems, in particular with the relation of superprocesses (conditioned on the total mass process) and Fleming-Viot processes. Special attention is paid to the case of processes living in the set of absolutely continuous measures (with respect to Lebesgue measure) and the related stochastic evolution equations. Sample path properties of super-Brownian motion are surveyed in Chapter 9, as the probability to charge a ball, properties of the support process, characterization of sets hit by super- Brownian motion (R-polarity), multiple points and intersection. Outlines of proofs are included in some cases as illustration of the ideas and techniques involved. Also some open questions are mentioned. This chapter provides an introduction to this rich subject for research. Examples of the construction of measure-valued processes with interaction and additional spatial structure are given in Chapter 10. This is based on the Cameron-Martin-Girsanov formula and the so-called Feynman-Kac dual representation starting with the building blocks of measure-valued processes, namely superprocesses and Fleming-Viot processes. This chapter demonstrates the power of the combination of the martingale problem techniques and either stochastic calculus or duality. In Chapter 11 some powerful techniques are collected as an infinite particle representation of the Fleming-Viot process and a cluster representation for superprocesses. The final chapter is devoted to the family structure and is also intended as an introduction to the rapidly developing theory of historical respectively genealogical processes, which has already led to the solution of several unsolved problems of the theory of measure-valued processes. This Chapter 12 culminates in a complete probabilistic description of the genealogical structure of a population at a fixed time in terms of an infinite random tree for both the continuous superprocess and the Fleming-Viot process as well as the fact that the laws of the resulting infinite random trees are mutually singular. The list of references on the subject contains about 340 items.

For the entire collection see [Zbl 0778.00027].

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MSC:

60J80	Branching processes (Galton-Watson, birth-and-death, etc.)
60-02	Research exposition (monographs, survey articles) pertaining to probability theory
60G57	Random measures

Keywords:

Fisher-Wright; Hausdorff measure; infinite divisibility; coalescing; look-down process; McKean-Vlasov; exchangeability; occupation time; binary branching; Campbell measure; carrying dimension; collision local time; de Finetti cluster representation; Palm distribution; range of super-Brownian motion; multilevel; clumping; stepping stone model; nonstandard model; tightness