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Comparing the relative volume with a revolution manifold as a model. (English)

Zbl 0794.53036
Isr. J. Math. 84, No. 3, 385-401 (1993).

Given a pair (P, M), where M is an n-dimensional connected compact Riemannian manifold and P is a connected compact hypersurface of M, the relative volume of (P, M) is the quotient volume(P)/volume(M). In this paper the authors give a comparison theorem for the relative volume of such a pair, with some bounds on the Ricci curvature of M and the mean curvature of P, with respect to that of a model pair $(\mathcal{P}, \mathcal{M})$ where $\mathcal{M}$ is a revolution manifold and $\mathcal{P}$ a "parallel" of $\mathcal{M}$.

From *P. Berard* and *S. Gallot* [Semin. Gaulaouic-Meyer- Schwartz, Equations Dériv. Partielles 1983-1984, Exp. No. 15, 34 p. (1984; Zbl 0542.53025)] and *S. Gallot* [On the geometry of differentiable manifolds, Workshop, Rome/Italy 1986, Astérisque 163- 164, 31-91 (1988; Zbl 0674.53001)], a compact revolution manifold is a C^∞ (compact) Riemannian manifold $(M^\varphi; \langle\ ,\ \rangle)$ of dimension n for which there are two points $\mathcal{N}$ and $\mathcal{S}$ of M^φ, a real number $L > 0$, a function $\varphi : [0, 2L] \to [0, +\infty)$ and a diffeomorphism $f : (0, 2L) \times S^{n-1} \to M - \{\mathcal{N}, \mathcal{S}\}$ such that at each $(s, x) \in (0, 2L) \times S^{n-1}$ we have $f^*\langle\ ,\ \rangle = ds^2 + \varphi^2(s)\langle\ ,\ \rangle_{S^{n-1}}$. A compact revolution manifold M^φ is symmetric if the curvature $\varphi(t)$ is symmetric with respect to L (i.e. $\rho(2L-t) = \rho(t)$, or, equivalently, $\rho(L-t) = \rho(L+t)$). When M^φ is symmetric it is said that M^φ is lengthened if ρ is decreasing on the interval $[0, L]$, and M^φ is called flattened if ρ is increasing on $[0, L]$. The main result:

Theorem 1.6. Let M^φ be a compact lengthened convex revolution manifold with sectional curvature $\rho(t)$. Let $R \in [0, L]$, and let M and P be as before. If $\mathrm{Ric}(\gamma_p'(t), \gamma_p'(t)) \geq (n-1)\rho(R-t)$ for every t such that $-c(-N(p)) \leq t \leq c(N(p))$, $|H| \leq \varphi'(R)/\varphi(R)$, then $v(P, M) \geq v(S_R^\varphi, M^\varphi)$. Moreover, the equality implies that there is an isometry between M and M sending P onto S_R. A similar theorem when M is a compact flattened convex revolution manifold, if this exists, is not proved so far.

Reviewer: I.Pop (Iaşi)

MSC:

53C40 Global submanifolds
53C20 Global Riemannian geometry, including pinching

Keywords:

relative volume; comparison theorem; Ricci curvature; mean curvature

Full Text: DOI

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