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Measurability properties of sets of Vitali's type. (English) Zbl 0795.28010

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Let G be a group acting on a fixed set X and μ a G-invariant countably additive measure on X. If H is a subgroup of G, then H- selector means a set having exactly one point in common with each orbit of H. The action of G is μ-free if $\mu^*(\{x \in X : hx = x\}) = 0$ for any $h \in G\backslash\{e\}$ ($e =$ the identity of G). The cardinality of a set A is denoted by $|A|$. Also for a cardinal number λ, $cf(\lambda) = \min\{K : K$ is an ordinal and $\exists f : K \to \lambda$, $\lambda = \bigcup_{\alpha<K} f(\alpha)\}$. The following two theorems have been proved.

Theorem 1. Let G be uncountable and let μ be σ-finite. Suppose G acts μ-freely on X. Then there exists a countable subgroup H of G such that each H-selector is nonmeasurable with respect to any invariant extension of μ. Theorem 2. Assume $cf(|G|) > \omega$. Suppose also that G acts freely on X. Let μ be σ-finite and ergodic. Then there exists an invariant extension $\bar{\mu}$ of μ such that for each subgroup H of G with $|H| = |G|$ there is a $\bar{\mu}$-measurable H-selector.

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MSC:

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