Ludwig, Jean; **Zahir, Hamid**

On the nilpotent $*$-Fourier transform. (English) Zbl 0798.22004

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For a connected and simply connected nilpotent Lie group G, one considers the Fourier transform $\mathcal{E}$: $\mathcal{S}(G) \to C^\infty(V, \mathcal{S}(\mathbb{R}^{2d}))$:

$$\mathcal{E}(f)(\lambda, p, q) = \int_G e^{-ia(g,\lambda,p,q)} f(g)dg,$$

where $\mathcal{S}(G)$ is the Schwartz space of G, $V \times \mathbb{R}^{2d} \to \mathcal{O}$ an adapted diffeomorphic parametrization for a G-invariant Zariski open set $\mathcal{O}$ on the dual space g^* of the Lie algebra $g = \mathrm{Lie}(G)$ and $a(.,.,.,.)$ a real function, polynomial in p and q and rational in λ, with singularities outside V. The authors prove the surjectivity of this transformation (Corollary 2.2.4) and therefore extend this Fourier transform to distributions. This leads them in particular to have some interesting results, for example $\mathcal{E}(\delta_x) = e^{ia(.,.,.,.)}$, it transforms the convolution product on group G to the Moyal $*$-product on $V \times \mathbb{R}^{2d}$, $\mathcal{E}(T * f) = \mathcal{E}(T) * \mathcal{E}(f)$.

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MSC:

22E27 Representations of nilpotent and solvable Lie groups (special orbital integrals, non-type I representations, etc.)
43A30 Fourier and Fourier-Stieltjes transforms on nonabelian groups and on semigroups, etc.

Keywords:

simply connected nilpotent Lie group; Fourier transform; Schwartz space; diffeomorphic parametrization; Lie algebra; distributions; convolution product; Moyal $*$-product

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