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Decompositions of rational convex polytopes. (English) Zbl 0812.52012

Ann. Discrete Math. 6, 333-342 (1980).

Let $\mathcal{P}$ be a rational d-polytope (its vertices have rational Cartesian coordinates), $i(\mathcal{P}, n)$ the number of lattice points (with integer coordinates) in $\mathcal{P}$, and $J(\mathcal{P}, \lambda) = 1 + \sum_{n \geq 1} i(\mathcal{P}, n)\lambda^n$ the corresponding generating function. The function $J(\mathcal{P}, \lambda)$ has been much investigated [see *E. Ehrhardt*, Polynômes arithmétiques et méthode des polyèdres en combinatoire, Birkhäuser, Basel (1977; Zbl 0337.10019)]; here the author develops further properties. For example, if $\mathcal{P}$ is a lattice polytope, then $J(\mathcal{P}, \lambda) = W(\mathcal{P}, \lambda)/(1 - \lambda)^{d+1}$, where $W(\mathcal{P}, \lambda)$ is a polynomial of degree at most d with nonnegative integer coefficients (the proof is more geometrical than that of the author [Duke Math. J. 43, No. 3, 511- 531 (1976; Zbl 0335.05010)]); in certain (described) circumstances, these coefficients are simple functions of the numbers of faces of $\mathcal{P}$. In general, $i(\mathcal{P}, n)$ is a near polynomial ("polynôme mixte") in n, whose coefficients vary cyclically; the author verifies a conjecture of Ehrhart about when these coefficients are fixed (a proof in a more general situation was given by the reviewer [Arch. Math. 31, 509-516 (1978; Zbl 0395.52006)]).

For the entire collection see [Zbl 0435.00003].

MSC:

52C22	Tilings in n dimensions (aspects of discrete geometry)
05B45	Combinatorial aspects of tessellation and tiling problems
52B20	Lattice polytopes in convex geometry (including relations with commutative algebra and algebraic geometry)

Keywords:

lattice polytope; conjecture of Ehrhart

Full Text: DOI