Vaisman, Izu

Lectures on the geometry of Poisson manifolds. (English) Zbl 0810.53019

Progress in Mathematics (Boston, Mass.). 118. Basel: Birkhäuser. vi, 205 p. (1994).

A long overdue systematic self-contained presentation of the subject. The book presents the basic theory including most results of the fundamental article by *A. Weinstein* [J. Differ. Geom. 18, 523-557 (1983; Zbl 0524.58011)] and outlines some of the further developments. In principle the only prerequisite is calculus on manifolds, however for understanding background and purpose of the theory some familiarity with symplectic geometry and its application is necessary.

Beginning with an invariant definition of the Schouten-Nijenhuis bracket fundamental properties of Poisson manifolds are developed: the symplectic foliation, the fundamental splitting, regularity and Poisson connections. The standard examples linear Poisson structure on $\mathbb{R}^n$ and Lie Poisson structure on the dual Lie algebra of Lie groups and coadjoint orbits and the embedding into an almost symplectic manifold with Dirac brackets are presented. A chapter on Poisson calculus includes contravariant exterior differentiation and graded Lie algebra structures on Poisson manifolds. Chapters on Poisson cohomology and its spectral sequence, quantization and deformations, a discussion of Poisson morphisms, group actions, momenta and reduction of Poisson manifolds follow. The final three chapters deal with symplectic realizations, realizations by symplectic groupoids and with Poisson-Lie groups.

All major results include complete proofs, however no physical applications are given. The book serves well as an introduction and an overview of the subject and a long list of references helps with further studies.

Reviewer: C.Günther (Libby)

MSC:

53C15	General geometric structures on manifolds (almost complex, almost product structures, etc.)
37J99	Dynamical aspects of finite-dimensional Hamiltonian and Lagrangian systems
53D50	Geometric quantization
53-02	Research exposition (monographs, survey articles) pertaining to differential geometry

Keywords:

Schouten-Nijenhuis bracket; Poisson manifolds; symplectic foliation; fundamental splitting; Poisson connections; graded Lie algebra structures; Poisson cohomology; quantization; Poisson morphisms; symplectic groupoids; Poisson-Lie groups; bibliography