

Lê Tuấn Hoa; Vogel, Wolfgang**Castelnuovo-Mumford regularity and hyperplane sections.** (English) Zbl 0816.14023

J. Algebra 163, No. 2, 348-365 (1994).

Let $X \subset \mathbb{P}^n$ be a projective variety and \mathcal{I}_X the ideal sheaf of X in \mathbb{P}^n . One says that X is k -regular if $H^i(\mathbb{P}^n, \mathcal{I}_X(k-i)) = 0$ for all $i \geq 1$. The Castelnuovo-Mumford regularity $\text{reg}(X)$ of $X \subset \mathbb{P}^n$ is the last such k . Bounds on the regularity of X are important in connection with algorithms for computing syzygies and this accounts for some of the recent interest in this question.

There is a classical technique aimed at bounding $\text{reg}(X)$ by studying $H^i(\mathbb{P}^n, \mathcal{I}_{X \cap H})$ for a hyperplane section $X \cap H$ of X . By using this method (corollaries 1 and 2.3), in the present paper the authors give a new bound for $\text{reg}(X)$ of arithmetically quasi-Buchsbaum varieties $X \subset \mathbb{P}^n$, in terms of their invariants (corollary 2). Recall that X is said to be arithmetically quasi-Buchsbaum if $\bigoplus_{p \in \mathbb{Z}} H^i(\mathbb{P}^n, \mathcal{I}_X(p))$ is annihilated by (x_0, \dots, x_n) for all $1 \leq i \leq \dim X$. The authors obtain this result in a general form (proposition 3.6), as special case of their investigation of (k, r) -Buchsbaum modules (see definition 1.1), from which they deduce other new bounds on the regularity, improve some old result and, in some case, provide better bounds as expected (e.g. theorems 3.4 and 3.8 and their example on p. 349).

Reviewer: V.Di Gennaro (Roma)

MSC:

- 14N05 Projective techniques in algebraic geometry
- 14C17 Intersection theory, characteristic classes, intersection multiplicities in algebraic geometry
- 14F17 Vanishing theorems in algebraic geometry
- 14M05 Varieties defined by ring conditions (factorial, Cohen-Macaulay, semi-normal)
- 13D02 Syzygies, resolutions, complexes and commutative rings

Cited in 1 Review
Cited in 4 Documents

Keywords:

hyperplane sections; Cohen-Macaulay varieties; local cohomology; arithmetically quasi-Buchsbaum variety; Buchsbaum modules; Castelnuovo-Mumford regularity; syzygies

Full Text: [DOI](#)