Berkovich, Vladimir G.

Vanishing cycles for formal schemes. (English) Zbl 0791.14008

Invent. Math. 115, No. 3, 539-571 (1994).

Let k be a non-Archimedean field, and let $\mathfrak{X}$ be a formal scheme locally finitely presented over the ring of integers k^0. In this work one constructs and studies the vanishing cycles functor from the category of étale sheaves on the generic fibres $\mathfrak{X}_\eta$ of $\mathfrak{X}$ (which is a k-analytic space) to the category of étale sheaves on the closed fibre $\mathfrak{X}_{\bar{s}}$ of $\mathfrak{X}$ (which is a scheme over the residue field of the separable closure of k). One proves that if $\mathfrak{X}$ is the formal completion $\hat{\mathcal{X}}$ of a scheme $\mathcal{X}$ finitely presented over k^0 along the closed fibre, then the vanishing cycles sheaves of $\hat{\mathcal{X}}$ are canonically isomorphic to those of $\mathcal{X}$ [as defined by *P. Deligne* in Sémin. Géométrie algébrique, 1967-1969, SGA7 II, Lect. Notes Math. 340, Exposé XIII, 82-115 (1973; Zbl 0266.14008)]. In particular, the vanishing cycles sheaves of $\mathcal{X}$ depend only on $\hat{\mathcal{X}}$, and any morphism $\varphi : \hat{\mathcal{Y}} \to \hat{\mathcal{X}}$ induces a homomorphism from the pullback of the vanishing cycles sheaves of $\mathcal{X}$ under $\varphi_{\bar{s}} : \mathcal{Y}_{\bar{s}} \to \mathcal{X}_{\bar{s}}$ to those of $\mathcal{Y}$. Furthermore, one proves that, for each $\hat{\mathcal{X}}$, there exists a nontrivial ideal of k^0 such that if two morphisms $\varphi, \psi : \hat{\mathcal{Y}} \to \hat{\mathcal{X}}$ coincide modulo this ideal, then the homomorphisms between the vanishing cycles sheaves induced by φ and ψ coincide. These facts were conjectured by P. Deligne.

The second fact is deduced from a theorem on the continuity of the action of the set of morphisms between two analytic spaces on their étale cohomology groups. Its particular case states the following. Let $X = \mathcal{M}(\mathcal{A})$ be a k-affinoid space, and let $f_1, \dots, f_n$ be a k-affinoid generating system of elements of $\mathcal{A}$. Then for any discrete $\mathrm{Gal}(k^s/k)$-module Λ and any element of $\alpha \in H^q(X, \Lambda)$ there exist $t_1, \dots, t_n > 0$ such that, for any pair of morphisms $\varphi, \psi : Y \to X$ over k with $\max_{y \in Y} |(\varphi^* f_i - \psi^* f_i)(y)| \leq t_i$, $1 \leq i \leq n$, one has $\varphi^*(\alpha) = \psi^*(\alpha)$ in $H^q(Y, \Lambda)$. The essential ingredient of the proof is a generalization of the classical Krasner lemma. This result implies, in particular, the following fact. If a k-analytic group G acts on a k-analytic space X, then the étale cohomology groups of X with compact support are discrete $G(k)$-modules. The present paper is based on the previous works of the author ["Spectral theory and analytic geometry over non-Archimedean fields", Math. Surveys Monographs 33 (1990; Zbl 0715.14013) and "Étale cohomology for non-Archimedean analytic spaces", Publ. Math., Inst. Hautes Étud. Sci. 78, 5-171 (1993)].

Reviewer: V.G.Berkovich (Rehovot)

MSC:

14F20	Étale and other Grothendieck topologies and (co)homologies
14F99	(Co)homology theory in algebraic geometry
18F20	Presheaves and sheaves, stacks, descent conditions (category-theoretic aspects)
14G20	Local ground fields in algebraic geometry
14C25	Algebraic cycles

Keywords:

analytic group; non-Archimedean field; formal scheme; vanishing cycles functor; étale sheaves

Full Text: DOI EuDML

References:

Ber1 Berkovich, V. G.: Spectral theory and analytic geometry over non-Archimedean fields, Mathematical Surveys and Monographs, vol. 33, American Mathematical Society, 1990

Ber2 Berkovich, V. G.: Étale cohomology for non-Archimedean analytic spaces, Publ. Math. IHES78, 5-161 (1994)

Ber3 Berkovich, V. G.: Vanishing cycles for non-Archimedean analytic spaces, (submitted to Journal of the AMS)

BGR Bosch, S.; Güntzer, U.; Remmert, R.: Non-Archimedean analysis. A systematic approach to rigid analytic geometry, Grundlehren der Mathematischen Wissenschaften, Bd. 261, Springer, Berlin-Heidelberg-New York, 1984 · Zbl 0539.14017

EGA1 Grothendieck, A.; Dieudonné, J.: Eléments de Géométrie Algébrique. I. Le langage des schémas, Springer, Berlin-Heidelberg-New York, 1971

God Godement, R.: Topologie Algébrique et Théorie des Faisceux, Hermann, Paris, 1958

Kel Kelley, J. L.: General Topology, D. Van Nostrand Company, Toronto-London-New York, 1957

Ray Raynaud, M.: Anneaux Locaux Henséliens, Lecture Notes in Math.169, Springer, Berlin-Heidelberg-New York, 1970

SGA1 Grothendieck, A.: Seminaire de Géométrie Algébrique. I. Revêtements étales et Groupe Fondemental, Lecture Notes in Math.224, Springer, Berlin-Heidelberg-New York, 1971

SGA4 Artin, M., Grothendieck, A., Verdier, J.-L.: Théorie des Topos et Cohomologie Étale des Schémas, Lecture Notes in Math.269, 270, 305, Springer, Berlin-Heidelberg-New York, 1972-1973

SGA41/2 Deligne, P. et al.: Cohomologie Étale, Lecture Notes in Math.569, Springer, Berlin-Heidelberg-New York, 1977 · Zbl 0349.14008

SGA7 Grothendieck, A., Deligne, P., Katz, N.: Groupes de Monodromie en Géométrie Algébrique, Lecture Notes in Math.288, 340 Springer, Berlin-Heidelberg-New York, 1972-1973