**Farkas, Daniel R.**; **Snider, Robert L.**

**Simple augmentation modules.** (English) Zbl 0802.20006

Q. J. Math., Oxf. II. Ser. 45, No. 177, 29-42 (1994).

Let $k$ be a field, $G$ be a group, $k[G]$ be the group algebra of the group $G$ over the field $k$. If $X$ is a set, $G$ is a permutation group on $X$, then $k[X]$ is the $k$-vector space with $X$ as a basis, it becomes a $k[G]$-module by extending the action of $G$ on $X$ in the obvious way. Put $\omega_k(X) = \{\sum_{x\in X} \lambda_x x \mid \sum_{x\in X} \lambda_x = 0\}$. The submodule $\omega_k(X)$ is called the augmentation module.

The basic results here are Theorem 9. Let $X$ be an infinite set, $G$ be a permutation group on $X$ and $\omega_k(X)$ is a simple $k[G]$- module. Then $X \setminus \{y\}$ has no finite $\mathrm{Stab}_G(y)$- orbits for any $y \in X$. Theorem 11. Suppose $G$ acts effectively on the infinite set $X$ and $\omega_k(X)$ is a simple $k[G]$-module. Then $\mathrm{FC}\{G\} = \{x \in G \mid |G : C_G(x)| \text{ is finite}\} = \langle 1\rangle$. Theorem 13. Let $G$ be a group, $A$ be a nonidentity, normal, torsion free abelian subgroup of finite rank. Suppose that $G$ acts effectively on $X$. Then $\omega_k(X)$ is a simple $k[G]$-module if and only if char $k > 0$ and no intermediate subgroup of $A$ has a finite $G$-orbit.

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**MSC:**

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|---|---|
| 20C07 | Group rings of infinite groups and their modules (group-theoretic aspects) |
| 16S34 | Group rings |
| 20B07 | General theory for infinite permutation groups |
| 16D60 | Simple and semisimple modules, primitive rings and ideals in associative algebras |



**Keywords:**

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