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Closed extremals on two-dimensional manifolds. (English. Russian original) ⟨Zbl 0791.58026⟩

Russ. Math. Surv. 47, No. 2, 163-211 (1992); translation from Usp. Mat. Nauk 47, No. 2(284), 143-185 (1992).

The aim of this survey is to present basic results and methods of the theory of closed extremals on two-dimensional manifolds from its beginning at the turn of the century until now.

In the first chapter the problem of existence of closed geodesics on two-dimensional manifolds is discussed. This chapter exposes the recent situation of the Lyusternik-Shnirel'man theorem whose complete proofs were presented in papers of *W. Ballmann* [Bonn. Math. Schrift. 102, 1–25 (1978; Zbl 0394.53027)], *M. Grayson* [Ann. Math. (2) 129, No. 1, 71–111 (1989; Zbl 0686.53036)], and the author [Russ. Acad. Sci., Izv., Math. 40, No. 3, 565–590 (1993); translation from Izv. Ross. Akad. Nauk, Ser. Mat. 56, No. 3, 605–635 (1992; Zbl 0771.53027)] and the Bangert-Franks theorem on the existence of infinitely many closed geodesics on manifolds homeomorphic to the two-dimensional sphere whose proof was obtained recently.

The second chapter is devoted to the periodic problem for not everywhere positive or multi-valued functionals. The study of this problem was begun by *S. P. Novikov* [Russ. Math. Surv. 37, No. 5, 1–56 (1982); translation from Usp. Mat. Nauk 37, No. 5(227), 3–49 (1982; Zbl 0571.58011)] in the early 80's when he tackled the problem of the existence of periodic motions of a particle in a magnetic field on a Riemannian manifold. This model example corresponds to many important problems of mathematical physics and analytical mechanics. In this case the difference from the length functional is obvious, it is the term linear in velocities that should be added to the Lagrange function of the length functional. If a magnetic field, i.e., a closed 2-form, is not exact, as in the case of the Dirac monopole, this linear part is multi-valued but the variational derivative correctly defines a closed 1-form on the space of closed smooth curves on the manifold under study. This chapter exposes the results of Novikov and the author obtained by appropriately extending the classical Morse theory, and also the results of Arnol'd, Kozlov, and Ginzburg obtained using symplectic geometry.

The theory of closed geodesics on convex surfaces is discussed in the third chapter exposes the development of ideas stemming from Poincaré's famous paper, a sketch of the Calabi-Cao theorem on the non-self-intersectedness of a shortest closed geodesic on a convex surface, and some other results.

The appendix is devoted to the recent situation in the general theory of closed geodesics.

It is obligatory to improve one mistranslation: Namely, the English version claims that the proof of Grayson is not clear whereas the Russian original reads: "Grayson's reasonings are rather involved, and the very deformation is weakly controllable. Perspicuity of the proof is also hindered by the fact that it leans heavily on non-constructible theorems of the existence and uniqueness of solutions of non-linear equations".

Reviewer: I. A. Taĭmanov

MSC:

58E10	Variational problems in applications to the theory of geodesics (problems in one independent variable)
58E05	Abstract critical point theory (Morse theory, Lyusternik-Shnirel'man theory, etc.) in infinite-dimensional spaces
58-02	Research exposition (monographs, survey articles) pertaining to global analysis
01A60	History of mathematics in the 20th century
58E15	Variational problems concerning extremal problems in several variables; Yang-Mills functionals

Cited in **3** Reviews
Cited in **19** Documents

Keywords:

survey; theory of closed extremals; two-dimensional manifolds; closed geodesics; Lyusternik-Shnirel'man

theorem; periodic problem; Morse theory; symplectic geometry