

**Kenig, Carlos E.; Ponce, Gustavo; Vega, Luis**

**Well-posedness and scattering results for the generalized Korteweg-de Vries equation via the contraction principle.** (English) [Zbl 0808.35128](#)

*Commun. Pure Appl. Math.* 46, No. 4, 527-620 (1993).

The well-posedness, which includes existence, uniqueness, and continuous dependence upon initial data, is studied for the initial-value problem for the generalized Korteweg-de Vries (KdV) equations of the form

$$u_t + u_{xxx} + u^k u_x = 0, \quad (1)$$

where  $k$  is a positive integer. It is well known that equation (1) is exactly integrable for  $k = 1$  (the classical KdV equation) and for  $k = 2$  (the modified KdV equation). At any value of  $k$ , equation (1) has two fundamental integrals of motion, the “momentum”  $\int_{-\infty}^{+\infty} u^2(x) dx$ , and the energy (Hamiltonian). These two integrals of motion are essentially used in the well-posedness analysis. The analysis is based on global estimates for an explicit solution of the linear initial-value problem associate to equation (1),  $v_t + v_{xxx} = 0$  combined with the contraction mapping principle. Introducing special functional norms, the authors demonstrate that, for each particular value of  $k$ , there is a relevant class of the Sobolev space to which the initial data  $u_0(x) \equiv u(x, t = 0)$  must belong in order to allow for the well-posedness proof, the existence being proved for finite times. The technique of the proof and the particular results obtained are essentially different for the cases  $k < 4$ ,  $k = 4$ , and  $k > 4$ , which reflects the fundamental property of equation (1): while at  $k < 4$  the solution developing from generic initial data remains smooth indefinitely long, the weak and strong collapse sets in at a finite time, respectively, at  $k = 4$  and at  $k > 4$ .

Reviewer: [B.A.Malomed \(Ramat Aviv\)](#)

**MSC:**

- [35Q53](#) KdV equations (Korteweg-de Vries equations)
- [35P25](#) Scattering theory for PDEs
- [35A05](#) General existence and uniqueness theorems (PDE) (MSC2000)
- [35B45](#) A priori estimates in context of PDEs

Cited in **10** Reviews  
Cited in **539** Documents

**Keywords:**

[well-posedness of KdV equations](#); [collapse](#); [Sobolev spaces](#)

**Full Text:** [DOI](#)

**References:**

- [1] Benedek, Proc. Nat. Acad. Sci. USA 48 pp 356– (1962)
- [2] Benjamin, J. Fluid Mech. 29 pp 559– (1967)
- [3] and , Interpolation Spaces, Springer-Verlag, New York and Berlin, 1970.
- [4] Blasco, Studia Math. 92 pp 205– (1989)
- [5] Bona, Duke Math. J. 43 pp 87– (1976)
- [6] Bona, Proc. Roy. Soc. London A 278 pp 555– (1978)
- [7] Bona, Proc. Roy. Soc. London A 411 pp 395– (1987)
- [8] Bony, Ann. Scient. ENS 4 ème Série 14 pp 209– (1981)
- [9] A remark on Schrödinger operators, preprint.
- [10] Some analytical problems related to statistical mechanics, Euclidean Harmonic analysis, Lecture Notes in Math No. 779, Springer-Verlag, Berlin and New York, 1979, pp. 9–45.
- [11] Cazenave, Nonlinear Anal. TMA 14 pp 807– (1990)
- [12] Christ, J. Funct. Anal. 100 pp 87– (1991)
- [13] Cohen, Duke Math. J. 45 pp 149– (1978)

- [14] Cohen, *SIAM J. Math. Anal.* 18 pp 991– (1987)
- [15] and , *Au-delà des Opérateurs Pseudodifférentiels*, Astérisque 57, Société Mathématique de France, 1973.
- [16] and , *Nonlinear harmonic analysis, operator theory and P. D. E.*, pp. 3–45 in: *Beijing Lectures in Harmonic Analysis*, Princeton University Press, 1986.
- [17] Coifman, *J. Funct. Anal.* 62 pp 304– (1985)
- [18] Constantin, *J. Amer. Math. Soc.* 1 pp 413– (1988)
- [19] Craig, *J. Diff. Eqs.* 87 pp 38– (1990)
- [20] Craig, *Ann. IHP Analyse Nonlinéaire* 9 pp 147– (1992)
- [21] and , *A note on the almost everywhere behavior of solutions to the Schrödinger equation*, pp. 205–208 in: *Harmonic Analysis*, Lecture Notes in Math. No. 908, Springer-Verlag, Berlin and New York, 1982.
- [22] Dubrovin, *Russian math. Surveys* 31 pp 59– (1976)
- [23] Fefferman, *Amer. J. Math.* 93 pp 107– (1971)
- [24] Fefferman, *Acta Math.* 129 pp 137– (1972)
- [25] and , *Weighted Norm Inequalities and Related Topics*, North-Holland, Amsterdam-Oxford-New York, 1985.
- [26] Gardner, *Phys. Rev. Letters* 19 pp 1095– (1967)
- [27] Gardner, *Comm. Pure Appl. Math.* 27 pp 97– (1974)
- [28] Ginibre, *J. Math. Pure Appl.* 64 pp 363– (1985)
- [29] Ginibre, *Comm. Math. Phys.* 144 pp 163– (1992)
- [30] Ginibre, *SIAM J. Math. Anal.* 20 pp 1388– (1989)
- [31] Glassey, *J. Math. Phys.* 18 pp 1794– (1979)
- [32] Global existence of small analytic solutions to nonlinear Schrödinger equations II, preprint.
- [33] Quasilinear equations of evolutions, with applications to partial differential equations, pp. 27–50 in: *Lecture Notes in Math.* No. 448, Springer-Verlag, New York, 1975.
- [34] Kato, *Manuscripta Math* 29 pp 89– (1979)
- [35] Kato, *Studies in Applied Math.* 8 pp 93– (1983)
- [36] Kato, *Comm. Pure Appl. Math.* 41 pp 891– (1988)
- [37] Kappeler, *Comm. PDE* 11 pp 927– (1986)
- [38] Kenig, *Duke Math. J.* 59 pp 585– (1989)
- [39] Kenig, *Indiana Univ. Math. J.* 40 pp 33– (1991)
- [40] Kenig, *J. Amer. Math. Soc* 4 pp 323– (1991)
- [41] , and , *On the generalized Benjamin-Ono equation*, *Trans. Amer. Math. Soc.*, to appear. · [Zbl 0804.35105](#)
- [42] , and , *Small solutions to nonlinear Schrödinger equations*, *Ann. IHP*, to appear. · [Zbl 0786.35121](#)
- [43] Kenig, *Trans. Amer. Math. Soc.* 230 pp 239– (1983)
- [44] Kichenassamy, *SIAM J. Math. Anal.* 23 pp 1141– (1992)
- [45] Korteweg, *Philos. Mag.* 5 39 pp 422– (1895) · [doi:10.1080/14786449508620739](#)
- [46] Kruzhkov, *Math. USSR Sb.* 48 pp 93– (1984)
- [47] Lax, *Comm. Pure Appl. Math.* 21 pp 467– (1968)
- [48] *Mixed norm estimates for the Klein-Gordon equation*, pp. 638–652 in: *Proceedings of a Conference on Harmonic Analysis*, Chicago, Wardsworth Math. series, 1981.
- [49] Merle, *Comm. Math. Phys.* 129 pp 223– (1990)
- [50] *Ondelettes I*, Hermann, Paris, 1990.
- [51] *Ondelettes et Opérateurs II*, Hermann, Paris, 1990.
- [52] Miura, *SIAM review* 18 pp 412– (1976)
- [53] Miura, *J. Math. Phys.* 9 pp 1204– (1968)
- [54] Nawa, *Funkcialaj Ekvacioj* 32 pp 417– (1989)
- [55] *Hamiltonian and non-Hamiltonian models for water waves*, *Lecture Notes in Physics* No. 195, Springer-Verlag, 1984, pp. 273–290.
- [56] Ono, *J. Phy. Soc. Japan* 39 pp 1082– (1975)
- [57] Pecher, *Math. Z.* 185 pp 261– (1985)
- [58] Ponce, *J. Funct. Anal.* 90 pp 445– (1990)
- [59] Rubio de Francia, *Adv. Math.* 62 pp 7– (1988)
- [60] Sachs, *Comm. PDE* 10 pp 29– (1985)

- [61] Saut, J. *Math. Pures Appl.* 58 pp 21– (1979)
- [62] Saut, *Israel J. Math.* 24 pp 78– (1976)
- [63] Scott, *Proc. IEEE* 61 pp 1443– (1973)
- [64] Sjöberg, J. *Math. Anal. Appl.* 29 pp 569– (1970)
- [65] Sjölin, *Duke Math. J.* 55 pp 699– (1987)
- [66] Stein, *Trans. Amer. Math. Soc.* 83 pp 482– (1956)
- [67] *Singular Integrals and Differentiability Properties of Functions*, Princeton University Press, 1970.
- [68] Oscillatory integrals in Fourier analysis, pp. 307–355 in: *Beijing Lectures in Harmonic Analysis*, Princeton University Press, 1986.
- [69] and , *Introduction to Fourier Analysis in Euclidean Spaces*, Princeton University Press, 1971.
- [70] Strauss, J. *Funct. Anal.* 41 pp 110– (1981)
- [71] Strichartz, J. *Math. Mech.* 16 pp 1031– (1967)
- [72] Strichartz, *Duke Math. J.* 44 pp 705– (1977)
- [73] Tanaka, *Osaka J. Math.* 11 pp 49– (1974)
- [74] *Pseudo-Differential Operators and Nonlinear PDE*, Birkhäuser, Boston, 1991. · doi:10.1007/978-1-4612-0431-2
- [75] Temam, J. *Math. Pures Appl.* 48 pp 159– (1969)
- [76] Tomas, *Bull. AMS* 81 pp 477– (1975)
- [77] Tsutsumi, *Funkcialaj Ekvacioj* 30 pp 115– (1987)
- [78] Tsutsumi, *SIAM J. Math. Anal.* 20 pp 582– (1989)
- [79] El multiplicador de Schrödinger, la función maximal y los operadores de restricción, *Doctoral Thesis*, Universidad Autonoma de Madrid, 1988.
- [80] Vega, *Proc. Amer. Math. Soc.* 102 pp 874– (1988)
- [81] Weinstein, *Lectures Appl. Math.* 23 pp 23– (1986)
- [82] Weinstein, *Comm. Pure Appl. Math.* 39 pp 51– (1986)
- [83] Two-dimensional gravity and intersection theory on moduli space, preprint. · Zbl 0757.53049
- [84] A note in interpolation spaces, pp. 199–204 in: *Lecture Notes in Math.* No. 908, Springer-Verlag, New York, 1982.
- [85] Zakharov, *Sov. Phys. JETP* 34 pp 62– (1972)

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.