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**A nonlinear instability for  $3 \times 3$  systems of conservation laws.** (English) Zbl 0820.35093  
*Commun. Math. Phys.* 162, No. 1, 47-59 (1994).

**Summary:** The phenomenon of nonlinear resonance provides a mechanism for the unbounded amplification of small solutions of systems of conservation laws. We construct spatially  $2\pi$ -periodic solutions  $u^N \in C^\infty([0, t_N] \times \mathbb{R})$  with  $t_N$  bounded, satisfying  $\|u^N\|_{L^\infty([0, t_N] \times \mathbb{R})} \rightarrow 0$ ,  $\int_0^{2\pi} |\partial_x u^N(0, x)| dx \leq C$ ,  $\int_0^{2\pi} |\partial_x u^N(t_N, x)| dx \geq N$ ,  $\|u^N(t_N, x)\|_{L^p(\mathbb{R})} \geq N \|u^N(0, x)\|_{L^p(\mathbb{R})}$ ,  $1 \leq p \leq \infty$ . The variation grows arbitrarily large, and the sup norm is amplified by arbitrarily large factors.

**MSC:**

**35L65** Hyperbolic conservation laws

Cited in **10** Documents

**Keywords:**

blow up; nonlinear resonance

**Full Text:** [DOI](#)

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