Wada, Masaaki

Twisted Alexander polynomial for finitely presentable groups. (English) Zbl 0822.57006

Topology 33, No. 2, 241-256 (1994).

The usual definition of the Alexander polynomial of a link extends readily to a Laurent polynomial invariant for any epimorphism $\alpha : \Gamma \to Z^r$, where Γ is a finitely presentable group. This paper presents a further extension to an invariant depending also on a linear representation $\rho : \Gamma \to \mathrm{GL}(n, R)$, where R is a unique factorization domain. The resulting invariant $\Delta_{\Gamma,\rho}(t_1, \ldots, t_r)$ is a rational function in the quotient field of $R[t_1, \ldots, t_r]$, well defined up to multiplication by units of this ring. For example, if $\Gamma = Z$ (generated by t) and $\alpha = \mathrm{id}_Z$ then $\Delta_{\Gamma,\rho}(t) = \det(I - t\rho(t))^{-1}$. If $r > 1$ then $\Delta_{\Gamma,\rho}$ is a Laurent polynomial with coefficients in the field of fractions of R. In the final section twisted polynomials associated with representations into $\mathrm{GL}(2, Z/7Z)$ are used to distinguish the two 11 crossing knots with ordinary Alexander polynomial 1.

[Reviewer's remark. The general case can be subsumed into the special case $r = 0$ (α trivial), provided we assume that there is a generator whose image under ρ does not have 1 as an eigenvalue].

Reviewer: J.A.Hillman (Sydney)

MSC:

57M25	Knots and links in the 3-sphere (MSC2010)
57M05	Fundamental group, presentations, free differential calculus
20F05	Generators, relations, and presentations of groups

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