

**Scharlemann, Martin; Thompson, Abigail**

**Thin position and Heegaard splittings of the 3-sphere.** (English) Zbl 0820.57005  
*J. Differ. Geom.* 39, No. 2, 343-357 (1994).

The paper gives a clearly presented simple proof of Waldhausen's theorem, that a Heegaard splitting of  $S^3$  depends only on its genus. The main technique is to use the spine of one handlebody in the splitting, which is a trivalent graph, and manipulate it using edge slides, while keeping track of compressing discs in the complementary handlebody.

In the case of a splitting of  $S^3$ , the spine is put into a suitably minimal non-degenerate position relative to a height function; a series of lemmas then shows how to find either an unknotted cycle or a split link on the graph after edge slides. This allows a simple reduction of the splitting to one of lower genus, and an inductive proof of Waldhausen's theorem. Besides this application, the authors point to the lemmas as giving new characterizations of graphs in  $S^3$  whose complement is boundary-reducible.

As with other writings by these authors, the material is very well organized, and the combinatorial arguments follow so smoothly that their full power is not always immediately apparent.

Reviewer: [H.R.Morton \(Liverpool\)](#)

**MSC:**

57M25 Knots and links in the 3-sphere (MSC2010)  
57M15 Relations of low-dimensional topology with graph theory

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**Keywords:**

[graphs whose complement is boundary-reducible](#); [Waldhausen's theorem](#); [Heegaard splitting of  \$S^3\$](#) ; [height function](#); [unknotted cycle](#); [split link](#)

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