Shiga, Tokuzo

Two contrasting properties of solutions for one-dimensional stochastic partial differential equations. (English) Zbl 0801.60050

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The author considers the following equation

$$\partial u(t,x)/\partial t=\Delta u(t,x)+b\big(u(t,x)\big)+a\big(u(t,x)\big)\dot W(t,x),\ t\ge 0,\ x\in R,\quad u(0,x)=f(x), \tag{1}$$

where $\dot W(t,x)$ is a space-time white noise. Let

$$C_{\text{tem}}=\Big\{f\in C(R)\Big|\sup_{x\in R}\Big|\exp\{-\lambda|x|\}f(x)\Big|<\infty\quad\text{for every}\quad\lambda>0\Big\},$$

C^+_{tem} be the totality of nonnegative elements of C_{tem}, C^+_{c} be the totality of nonnegative, continuous functions with compact support.

The main results can be formulated as follows: Let $a(u)$, $b(u)$ be continuous functions, such that $|a(u)|+|b(u)|\le C(1+|u|)$, $u\in R$.

(i) If $a(0)=0$, $b(0)\ge 0$, then for every $f\in C^+_{\text{tem}}$ there exists a C^+_{tem}-valued solution $u(t,x)$ of problem (1).

(ii) Assume that for each $K>0$ there exists a constant $a_K>0$ such that $|a(u)|\ge a_K u^{1/2}$ for $0\le u\le K$, and that for some $C>0$: $|b(u)|\le C|u|$ for $u\in R$. Then if $f\in C^+_{\text{c}}$, $P\{u(t,x)\in C^+_{\text{c}}$ for every $t>0\}=1$ holds for every C^+_{tem}- valued solution of (1).

(iii) Let $a(u)$, $b(u)$ be Lipschitz continuous, and $u_1(t,x)$, $u_2(t,x)$ be two C_{tem}-valued solutions of (1) with the initial conditions $u_1(0)=f_1\in C_{\text{tem}}$ and $u_2(0)=f_2\in C_{\text{tem}}$. Suppose that $f_1\ge f_2$ and $f_1(x)>f_2(x)$ for some $x\in R$. Then $P\{u_1(t,x)>u_2(t,x)$ for every $t>0$ and every $x\in R\}=1$.

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MSC:

60H15 Stochastic partial differential equations (aspects of stochastic analysis)

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