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Intersections of algebraic and algebroid varieties. (English) Zbl 0063.00841

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Introduction: The object of this paper is to provide a local theory for the intersection multiplicities of algebraic varieties. The notion of intersection multiplicity of two algebraic varieties has been for the first time put on a solid base by van der Waerden. We consider the present step as an improvement on the van der Waerden theory for the following reasons:

(1) Esthetically, it seems natural to connect the multiplicity of a component M of the intersection of two varieties U and V with the local properties of V and V in the neighbourhood of M.

(2) Our theory includes an intersection theory for algebroid varieties.

(3) The theory of van der Waerden fails to attribute a multiplicity to a component M of the intersection of U and V in the case where, although M has the suitable dimension, some other components have too high dimensions.

The starting point of our considerations has been the observation that the multiplicity of the origin 0 in the intersection of two curves $f(X,Y)=0$, $g(X,Y)=0$ may be defined to be the degree of the field extension $K((X,Y))/K((f,g))$, where $K((X,Y))$ is the field of quotients of the ring of power series in X,Y with coefficients in the basic field K, and where $K((f,g))$ is the field of quotients of the ring of those power series in X,Y which can be expressed as power series in f and g. From there, I was led to the definition of multiplicity of a local ring with respect to a system of parameters, and then to the general notion of intersection multiplicity. In order to achieve this generalization, I have made extensive use of the notion of local ring, introduced by Krull.

This paper is divided into three parts. Part I contains some algebraic preparations. It is concerned with the study of the properties of a certain class of local rings, which I have called geometric local rings. The most important results in this first part are:

(1) what I have called the theorem of transition (§4, p. 22), because it is the tool by the use of which we can reduce questions of intersection multiplicities for algebraic varieties to similar questions for algebroid varieties;

(2) the associativity formula for multiplicities in local rings which is the source not only of the associativity formula for intersections, but of most other properties of intersection multiplicities as well.

Parts II and III are concerned with the intersection theory of algebroid and algebraic varieties respectively. Each of these parts begins with a short reminder of the main definitions. These are not meant to provide a first introduction to the notions with which algebraic geometry deals; their object is rather to determine unequivocally which one of the various possible points of view we adopt. In order to be able to reduce the theory for algebraic varieties to the corresponding theory for algebroid varieties, we show that an algebraic variety U splits up in the neighbourhood of one of its points into a certain number of algebroid varieties, which we call the sheets of U at the point.

Another intersection theory of algebraic varieties will be published shortly by A. Weil. I have been in constant communication with A. Weil during the writing of this paper; many of the ideas involved can be traced back to discussions of the subject between him and myself. It is therefore impossible for me to acknowledge with precision the extent of my indebtedness to him. Nevertheless, it can be said definitely that the statement of the "projection formula" and the knowledge of the fact that all properties of intersections can be derived from three basic theorems (namely, the theorem on intersection of product varieties, the projection formula and the formula of associativity) are both due specifically to A. Weil.

MSC:

14-XX Algebraic geometry

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