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A theory of covering and differentiation. (English) Zbl 0063.04111

Trans. Am. Math. Soc. 55, 205-235 (1944).

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Introduction. The isolation in 3.10 of a certain basic principle makes possible a covering theory which not only embraces every known theorem of the Vitali type but enables us to supplant the usual abstract differential theory with one more in accord with Lebesgue's. In fact, by substituting a new and frequently useful concept of regularity for the classical notion, we arrive at a general theory of differentiation which both includes and supplements similar results of Lebesgue.

We follow Banach's lead in paying special attention to nonadditive functions. By so doing a rather striking difference appears between the differential theory of interval functions on the one hand and that of set functions on the other.

As could be gathered from §2, we shall be concerned with a metric space $\mathcal{S}$, a fixed measure ϕ, and certain other measures. We presuppose such a knowledge of measure as might be derived from §3 of the subjoined paper by *J. F. Randolph* and myself (RM) [Trans. Am. Math. Soc. 55, 236–305 (1944; Zbl 0060.14002)]. Although this §3 of (RM) concerns itself with certain plane measures the reader should easily see that the results therein obtained hold equally well (with the obvious interpretations) when the plane is replaced by an abstract metric space. So extended, Theorems 3.7 and 3.13 of RM are used in the present paper (M). On the other hand Theorem 11.4 of (M) is used in proving Theorem 10.1 of (RM). This cross reference is noncircular since §3 of (RM) is entirely independent of (M).

MSC:

28Axx Classical measure theory

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