Havin, Victor; Jöricke, Burglind

The uncertainty principle in harmonic analysis. (English) Zbl 0827.42001

Ergebnisse der Mathematik und ihrer Grenzgebiete. 3. Folge. 28. Berlin: Springer-Verlag. xi, 543 p. (1994).

In the book under review, the Uncertainty Principle (UP) is stated as "it is impossible for a function and its Fourier transform to be simultaneously very small". According to the authors, the book is "a series of separate essays devoted to various concrete manifestations of the UP". They confine themselves to trigonometric series and integrals, so that "The Uncertainty Principle in Fourier Analysis" might be a better title. As they believe that the proof is more important than the theorem, they give several proofs of a number of assertions.

The book is divided into two parts, on real and complex methods respectively. In both parts, there are exact quantitative formulations of the UP, and examples indicating the limits to these theorems. Many results are included, and only a few are mentioned here, to give the flavour.

Examples of the "real variable" theorems include the F. and M. Riesz theorem, the Rudin-Carleson theorem, and the Ivashev-Musatov theorem. The first states that, if $\sum_{n\geq 0}\widehat{\mu}(n)e^{in\theta}$ is the Fourier series of a measure μ, then μ is absolutely continuous. The second states that, if A is a compact subset of the circle of measure 0, then any continuous function on A is the restriction to A of a continuous function f with Fourier series $\sum_{n\geq 0}\widehat{f}(n)e^{in\theta}$. The last shows that there exist singular measures μ with $\widehat{\mu}$ "very nearly" in $\ell^2(\mathbb{Z})$.

The "complex variable" part of the book includes a treatment of topics such as Hardy spaces, Blaschke products, and the Nevanlinna class. Some pairs of majorants (a,b), such that if $|f|\leq a$ and $|f|\leq b$, then $f=0$, are introduced. Results of Vol'berg and Borichev which show that, if $0\neq f\in L^1(\mathbb{T})$ and $\log|f|\in L^1(\mathbb{T})$, then $|\widehat{f}(n)|$ cannot decay very rapidly as $n\to\infty$, are presented. A delicate theorem of Beurling and Malliavin on measures whose Fourier transforms decay in prescribed ways illustrates the sharpness of such variants of the UP. The last of the usual forms of the UP is exemplified by another theorem of Beurling and Malliavin, on pairs (a,Λ) (where $a\in\mathbb{R}^+$ and $\Lambda\subset\mathbb{R}$) such that if T is a distribution on $\mathbb{R}$ supported in $(-a,a)$ and $\widehat{T}|_\Lambda=0$, then $T=0$. The final chapter deals with locality and non-locality for translation invariant operators, and ties the UP to the behaviour of solutions to differential equations.

The book presents in detail much material previously published only in Russian, or in journals. This reviewer found the language occasionally unusual (e.g., "parts" for "subsets", and "measures" are always positive), and would have welcomed more applications. All in all, it is useful and weighty addition to the literature.

Reviewer: M. G.Cowling (Kensington)

MSC:

42-02 Research exposition (monographs, survey articles) pertaining to harmonic analysis on Euclidean spaces
43-02 Research exposition (monographs, survey articles) pertaining to abstract harmonic analysis
42A38 Fourier and Fourier-Stieltjes transforms and other transforms of Fourier type
42B10 Fourier and Fourier-Stieltjes transforms and other transforms of Fourier type
42-01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to harmonic analysis on Euclidean spaces
30-02 Research exposition (monographs, survey articles) pertaining to functions of a complex variable

Keywords:

uncertainty principle; Fourier integrals; uniqueness theorem; Fourier transform; Fourier series