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On the growth of meromorphic functions of infinite order. (English) Zbl 0828.30013

J. Anal. Math. 64, 327-336 (1994).

For a meromorphic function f let $T(r,f)$ denote its Nevanlinna (or Ahlfors-Shimizu) characteristic and let $M(r,f)$ denote its maximum modulus. In this paper, it is proved that if γ is an increasing and differentiable function with $T(r,f) \leq \gamma(r)$ for large r, then

$$\liminf_{r\to\infty} \frac{\log M(r,f)}{r\gamma'(r)} \leq \pi$$

and that if ψ is a positive and continuously differentiable function such that $\psi(x)/x$ is nondecreasing, $\psi'(x) \leq \sqrt{\psi(x)}$, and $\int_{x_0}^{\infty} dx/\psi(x) < \infty$, then

$$\liminf_{r\to\infty} \frac{\log M(r,f)}{T(r,f)\psi(\log T(r,f))} = 0.$$

The proofs of these results are based on the method of Petrenko which was modified by *W. H. J. Fuchs* [Topics in Nevanlinna theory, Washington D.C., Proc. NRL Conf. Classical function theory, 1-32 (1970; Zbl 0294.30021)]. The proof in Fuchs' paper uses Pólya-peaks which exist in general only for meromorphic functions of finite lower order. For the proofs of the results in this paper the Pólya-peaks are replaced are replaced by a suitable other sequence of r-values which can be considered as Pólya-peaks of infinite order.

C. J. Dai, *D. Drasin* and *B. Q. Li* [J. Anal. Math. 55, 217-228 (1990; Zbl 0722.30016); Correction: J. Anal. Math. 57, 299-300 (1991; Zbl 0767.30027)] have shown that

$$\lim_{r\to\infty} \frac{\log M(r,f)}{T(r,f)\varphi(\log T(r,f))\log\varphi(\log T(r,f))} = 0$$

on a set of logarithmic density 1, where φ is an increasing, positive function with $\int_{x_0}^{\infty} \frac{dx}{\varphi(x)} < \infty$.

For a meromorphic function f and a complex number a let $b(a,f) := \liminf_{r\to\infty} \frac{\log M(r,1/(f-a))}{rT'(r,f)}$, $b(\infty,f) := \liminf_{r\to\infty} \frac{\log M(r,f)}{rT'(r,f)}$. The first result of this paper shows that $b(\infty,f) \leq \pi$ for functions of infinite order. Recently, A. Eremenko has proved that for every function f with lower order greater than 1/2 the set $\{a \in \widehat{\mathbb{C}};\, b(a,f) > 0\}$ is countable and that $\sum_{a\in\widehat{\mathbb{C}}} b(a,f) \leq 2\pi$.

Reviewer: G.Jank (Aachen)

MSC:

30D30	Meromorphic functions of one complex variable, general theory
30D35	Value distribution of meromorphic functions of one complex variable, Nevanlinna theory

Keywords:

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References:

[1] W. Bergweiler,Maximum modulus, characteristic, and area on the sphere, Analysis10 (1990), 163–176.Erratum: Analysis12 (1992), 67–69. · Zbl 0703.30025

[2] C. T. Chuang,Sur la croissance des fonctions, Kexue Tongbao26 (1981), 677–684. · Zbl 0496.30019

[3] C. J. Dai, D. Drasin and B. Q. Li,On the growth of entire and meromorphic functions of infinite order, J. Analyse Math.55

(1990), 217–228. Correction: J. Analyse Math.57 (1991), 299. · Zbl 0722.30016 · doi:10.1007/BF02789202

[4] W. H. J. Fuchs,Topics in Nevanlinna theory, inProceedings of the NRL Conference on Classical Function Theory, U.S. Government Printing Office, Washington, D.C., 1970, pp. 1–32.

[5] N. V. Govorov,The Paley conjecture, Funkcional. Anal. i. Prilozen3 (1969), 41–45.

[6] W. K. Hayman,Meromorphic Functions, Clarendon Press, Oxford, 1964.

[7] G. Jank and L. Volkmann,Einführung in die Theorie der ganzen und meromorphen Funktionen mit Anwendungen auf Differentialgleichungen, Birkhäuser, Basel-Boston-Stuttgart, 1985. · Zbl 0682.30001

[8] I. I. Marchenko and A. I. Shcherba,Growth of entire functions, Sib. Math. J.25 (1984), 598–605. · Zbl 0581.30025 · doi:10.1007/BF00968899

[9] R. Nevanlinna,Analytic Functions, Springer, New York-Heidelberg-Berlin, 1970. · Zbl 0199.12501

[10] V. N. Petrenko,Growth of functions of finite lower order, Math. USSR – Izvestija3 (1969), 391–432. · Zbl 0197.05303 · doi:10.1070/IM1969v003n02ABEH000786