

Terwilliger, Paul

A new inequality for distance-regular graphs. (English) Zbl 0814.05074
Discrete Math. 137, No. 1-3, 319-332 (1995).

Let Γ be a distance-regular graph with n vertices and diameter $d \geq 3$. Let i be fixed and $d(x, y) = i$. If $d(x, z) = 1$ then the possible values for $d(y, z)$ are $i, i+1, i-1$. The intersection numbers a_i, b_i, c_i denote the corresponding number of possible vertices z and are independent of the choice of x and y .

Let A be an $n \times n$ distance matrix and M a commutative (semi- simple) Bose-Mesner algebra, i.e. \mathbb{R} -algebra, generated by A . Let E be a primitive idempotent of M , $AE = EA = \theta E$ and $nE = \sum_0^d \theta_k^* A^k$.

An inequality, involving intersection numbers, $\theta \leq \theta_k^*$ is proved for each $3 \leq i \leq d$. It appears that equality is attained for $i = 3$ if and only if it is attained for all i and this happens if and only if Γ is a Q -polynomial with respect to E . The inequality looks simpler if for some q values $qc_i - b_i q(qc_{i-1} - b_{i-1})$ are independent of i . For $q \neq 0, 1, -1$ they can be written as $c_i \frac{q^{2-i} - 1}{1 - q^i} \geq c_{i-1}(q^{2-i} - 1)$.

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MSC:

05E30 Association schemes, strongly regular graphs

Cited in **36** Documents

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association scheme; distance-regular graph; intersection numbers; distance matrix; Bose-Mesner algebra; primitive idempotent; inequality; Q -polynomials

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