Terwilliger, Paul

A new inequality for distance-regular graphs. (English) Zbl 0814.05074

Discrete Math. 137, No. 1-3, 319-332 (1995).

Let Γ be a distance-regular graph with n vertices and diameter $d \geq 3$. Let i be fixed and $d(x,y) = i$. If $d(x,z) = 1$ then the possible values for $d(y,z)$ are i, $i+1$, $i-1$. The intersection numbers a_i, b_i, c_i denote the corresponding number of possible vertices z and are independent of the choice of x and y.

Let A be an $n \times n$ distance matrix and M a commutative (semi- simple) Bose-Mesner algebra, i.e. $\mathbb{R}$-algebra, generated by A. Let E be a primitive idempotent of M, $AE = EA = \theta E$ and $nE = \sum_0^d \theta_k^* A^k$.

An inequality, involving intersection numbers, $\theta \leq \theta_k^*$ is proved for each $3 \leq i \leq d$. It appears that equality is attained for $i = 3$ if and only if it is attained for all i and this happens if and only if Γ is a Q-polynomials with respect to E. The inequalities looks simpler if for some q values $qc_i - b_i q(qc_{i-1} - b_{i-1})$ are independent of i. For $q \neq 0, 1, -1$ they can be writen as $c_i \frac{q^{2-i}-1}{1-q^i} \geq c_{i-1}(q^{2-i} - 1)$.

Reviewer: V.Ufnarovski (Lund)

MSC:

05E30 Association schemes, strongly regular graphs

Keywords:

association scheme; distance-regular graph; intersection numbers; distance matrix; Bose-Mesner algebra; primitive idempotent; inequality; Q-polynomials

Full Text: DOI

References:

[1] Bannai, E.; Ito, T., Algebraic combinatories I: associations schemes, ()

[2] Brouwer, A.; Cohen, A.; Neumaier, A., Distance-regular graphs, (1989), Springer Menlo Park, CA · Zbl 0678.05038

[3] Cameron, P.; Goethals, J.; Seidel, J., The Krein condition, spherical designs, norton algebras, and permutation groups, Indag, math., 40, 196-206, (1978) · Zbl 0408.05016

[4] Neumaier, A., Krein conditions and near polygons, J. combin. theory ser. A, 54, 201-209, (1990) · Zbl 0738.05025

[5] Terwilliger, P., A characterization of \textit{P}- and Q-polynomial association schemes, J. combin. theory ser. A, 45, 8-26, (1987) · Zbl 0663.05016